MATH 221 QUIZ 5
OCTOBER 21, 2013

Solve the following two problems, showing all your work. Do not use theorems that haven't been covered in this class yet.

(1) Find the derivative of the following function:

$$f(x) = \tan\left(\sqrt{x^2 - 42}\right)$$

Using the chain rule,

$$\begin{aligned}
\frac{d}{dx}\tan\left(\sqrt{x^2-42}\right) &= \sec^2\left(\sqrt{x^2-42}\right)\frac{d}{dx}\sqrt{x^2-42} \\
&= \sec^2\left(\sqrt{x^2-42}\right)\frac{d}{dx}\left(x^2-42\right)^{1/2} \\
&= \sec^2\left(\sqrt{x^2-42}\right)\cdot\frac{1}{2}\left(x^2-42\right)^{-1/2}\frac{d}{dx}\left(x^2-42\right) \\
&= \sec^2\left(\sqrt{x^2-42}\right)\cdot\frac{1}{2}\left(x^2-42\right)^{-1/2}\cdot 2x \\
&= \frac{x}{\cos^2\left(\sqrt{x^2-42}\right)\cdot\sqrt{x^2-42}}.
\end{aligned}$$

(2) Let $f(x) = y$ be defined by the equation

$$\frac{xy}{x+y} = 1.$$

Use implicit differentiation to find $f'(x)$ in terms of x. (An expression for $f'(x)$ in terms of both x and y will only receive partial credit.)

Multiply both sides by $x + y$ to obtain $xy = x + y$. Implicitly differentiating, we have

$$\frac{d}{dx}(xy) = y + x\frac{dy}{dx}$$

and

$$\frac{d}{dx}(x+y) = 1 + \frac{dy}{dx},$$

so

$$\begin{aligned}
y + x\frac{dy}{dx} &= 1 + \frac{dy}{dx} \\
(x-1)\frac{dy}{dx} &= 1 - y \\
\frac{dy}{dx} &= \frac{1-y}{x-1}.
\end{aligned}$$

Using the original equation $xy = x + y$, we obtain $(x-1)y = x$, so $y = \frac{x}{x-1}$. Thus,

$$f'(x) = \frac{dy}{dx} = \frac{1-y}{x-1} = \frac{1-\frac{x}{x-1}}{x-1} = \frac{(x-1)-x}{(x-1)^2} = \frac{-1}{(x-1)^2}.$$