MATH 221 QUIZ 7
NOVEMBER 4, 2013

Solve the following problem, showing all your work.

Consider the function

$$f(x) = x^3 + \frac{1}{x^2}.$$

Find all (horizontal, vertical, or slant) asymptotes, zeros, stationary/critical points, inflection points, and any other information you need to accurately graph the function. Draw an accurate graph of the function, clearly labelling all the information you just found.

- Vertical asymptote at $x = 0$.
- No horizontal asymptotes:

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty}\left[x^3 + \frac{1}{x^2}\right] = \lim_{x\to\pm\infty} x^3 = \pm\infty.$$

- No slant asymptotes:

$$\lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty}\left[x^2 + \frac{1}{x^3}\right] = \lim_{x\to\pm\infty} x^2 = \infty.$$

- Zeros:

$$\begin{aligned} x^3 + \frac{1}{x^2} &= 0 \\ x^5 + 1 &= 0 \\ x^5 &= -1 \\ x &= -1. \end{aligned}$$

  (There are also four other complex-valued solutions to $x^5 = -1$, but we only care about the real-valued solution, $x = -1$.)
- First and second derivatives:

$$\begin{aligned} f'(x) &= 3x^2 - \frac{2}{x^3}, \\ f''(x) &= 6x + \frac{6}{x^4}. \end{aligned}$$

- Critical points:

$$\begin{aligned} 3x^2 - \frac{2}{x^3} &= 0 \\ 3x^5 - 2 &= 0 \\ x^5 &= \frac{2}{3} \\ x &= \sqrt[5]{\frac{2}{3}}. \end{aligned}$$

- Inflection points:

$$\begin{aligned} 6x + \frac{6}{x^4} &= 0 \\ 6x^5 + 6 &= 0 \\ x^5 &= -1 \\ x &= -1. \end{aligned}$$

- Local min/max: There's only one critical point, and it occurs at $x = \sqrt[5]{\frac{2}{3}} > 0$. Note that, for any $x > 0$,

$$f''(x) = 6x + \frac{6}{x^4} > 0.$$

  Thus, f is convex at the critical point, and so it is a local minimum.
- Convex or concave: check values of $f''(x)$ for, say, $x = -2$, $x = -0.5$, and $x = 1$ (or any other suitable values).