MATH 221 QUIZ 8
NOVEMBER 11, 2013

Solve the following two problems, showing all your work.

(1) Use l'Hôpital's rule to compute the following limit:

$$\lim_{x\to\pi/2} \frac{\cos(x)}{\sin(2x)}$$

Since $\cos\frac{\pi}{2} = 0$ and $\sin(\pi) = 0$, we can use l'Hôpital's rule:

$$\begin{aligned}
\lim_{x\to\pi/2} \frac{\cos(x)}{\sin(2x)} &= \lim_{x\to\pi/2} \frac{\frac{d}{dx}\cos(x)}{\frac{d}{dx}\sin(2x)} \\
&= \lim_{x\to\pi/2} \frac{-\sin(x)}{2\cdot\cos(2x)} \\
&= \frac{\lim_{x\to\pi/2}[-\sin(x)]}{\lim_{x\to\pi/2}[2\cdot\cos(2x)]} \\
&= \frac{-\sin\frac{\pi}{2}}{2\cdot\cos(2\cdot\frac{\pi}{2})} = \frac{-1}{2\cdot(-1)} = \frac{1}{2}.
\end{aligned}$$

(2) Consider the function

$$f(x) = \ln\left((\cos x)^2\right).$$

Determine the intervals on which the function is increasing and on which it is decreasing.

To find these intervals, we need to find the stationary points and the vertical asymptotes. Vertical asymptotes, if any, must occur due to the fact that $\lim_{x\searrow 0}\ln(x) = -\infty$. So, in particular, f has vertical asymptotes exactly when $(\cos x)^2 = 0$, that is, when $x = \frac{\pi}{2} + \pi k$ for some integer k.

To find stationary points, we need to find the derivative:

$$\begin{aligned}
f'(x) &= \frac{d}{dx}\ln\left((\cos x)^2\right) \\
&= \frac{1}{(\cos x)^2}\cdot\frac{d}{dx}(\cos x)^2 \\
&= \frac{1}{(\cos x)^2}\cdot 2(\cos x)\cdot\frac{d}{dx}(\cos x) \\
&= \frac{1}{(\cos x)^2}\cdot 2(\cos x)\cdot(-\sin x) \\
&= \frac{-2\sin(x)}{\cos(x)} = -2\tan(x).
\end{aligned}$$

So $f'(x) = 0$ exactly when $x = \pi k$ for some integer k.

Since f is increasing when $f'(x) = -2\tan(x) > 0$ and decreasing when $f'(x) = -2\tan(x) < 0$, it follows that f is increasing when $\tan(x) < 0$ and decreasing when $\tan(x) > 0$. Thus, f is increasing on the intervals $\left[-\frac{\pi}{2} + \pi k, \pi k\right]$ for each integer k, and decreasing on the intervals $\left[\pi k, \frac{\pi}{2} + \pi k\right]$ for each integer k.