

1. If $\gcd(a, b) = 1$ show that $\gcd(a + b, a - b) = 1$ or 2
2. In how many ways can 16 people be seated around two circular tables one that seats 10 while the other seats 6.
3. (a) How many arrangements are there of the letters in MISSISSIPPI?
(b) How many arrangements in part (a) have all the I's adjacent?
(c) How many have no consecutive S's?
4. Determine the coefficients of x^9y^3 in the expansion of
 - (a) $(x + y)^{12}$
 - (b) $(x + 2y)^{12}$
 - (c) $(2x - 3y)^{12}$
5. A high school football coach must select 11 seniors to play on the team. If he can do this in 12,376 ways, how many eligible seniors are there?
6. $\forall n \in \mathbb{Z}$ with $n \geq 0$
Prove that,

$$n^3 + (n + 1)^3 + (n + 2)^3 \text{ is divisible by } 9$$

7. Determine the probability p of each of the following events:
 - (a) An even number appear in the toss of a fair die.
 - (b) At least one tail appears in the toss of 3 fair coins.
 - (c) A white marble appears in the random drawing of 1 marble from a box containing 4 white, 3 red and 5 blue marbles.
8. Six married people are standing in a room. Two people are selected at random. What is the probability that,
 - (a) they are married?
 - (b) one is male the other female.
9. Suppose 5 marbles are placed in 5 boxes at random. Find the probability that exactly one of the boxes is empty.

10. Suppose A and B are events with $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$. Find the probability that,
- (a) A does not occur
 - (b) B does not occur
 - (c) A or B occurs
 - (d) Neither A nor B occurs

11. Prove that, for all $n \in \mathbb{N}$,

$$\sum_{r=1}^n (4r + 3) = 2n^2 + 5n$$

12. Determine the number of integer solutions to

$$x_1 + x_2 + x_3 + x_4 = 32$$

- (a) $x_i \geq 0, 1 \leq i \leq 4$
 - (b) $x_1, x_2 \geq 5, x_3, x_4 \geq 7$
 - (c) $x_1, x_2, x_3 > 0, 0 < x_4 \leq 25$
13. Determine how many ways 20 coins can be selected from four large containers filled with knickles, pennies, quarters and dimes.(each container only has one type of coin and has way more than 20 in each)
14. In how many ways can we place 24 books on four different shelves so that there is at least one book on each shelf?