Supp HW #4 (ie: practice exam)

1) Prove by contradiction that there are infinitely many primes

2) Find a counter-example to:
   - for any sets A & B $\overline{A \cup B} = \overline{A \cap B}$

3) Find infinitely many counter examples to:

   ~~[illegible]~~

   - $7K+2$ is a perfect sq

4) Prove ~~that~~ $\overline{A} \cup \overline{B} = \overline{A \cap B}$ & $\overline{A} \cap \overline{B} = \overline{A \cup B}$
   (ie: show DeMorgans law for sets)

5) (a) Assume $p \vee (q \wedge r)$ & $p \to s$
   Show $r \vee s$

   (b) Assume $\neg p \vee s$, $\neg t \vee (s \wedge r)$, $\neg q \vee r$, $p \vee q \vee t$
   Show $r \vee s$

6) Assume $\forall x\,[P(x) \vee q(x)]$
   & $\forall x\,[(\neg P(x) \wedge q(x)) \to r(x)]$

   Show $\forall\,[\neg r(x) \to P(x)]$

7) Show $\forall K \in \mathbb{N}$

$$K^2 = \binom{K}{2} + \binom{K+1}{2}$$

13) Prove that if any 10 pts are placed in an equilateral triangle of side length 1 $\Rightarrow$ at least two are at most $1/3$ apart.

8) let $n \in \mathbb{N}$

if $a_1, \ldots, a_n \in \mathbb{N}$ & $b_1, \ldots, b_n \in \mathbb{N}$

& $\forall\ 1 \leq i \leq n \quad a_i | b_i$

Show $(a_1 \cdot a_2 \cdot a_3 \cdots a_n) | (b_1 \cdot b_2 \cdots b_n)$

9) Construct a bijection ~~between~~

$b: \mathbb{N} \to B$

where $B = \{x_1 x_2 x_3 \cdots x_n \mid x_i = 1 \text{ or } 0\}$

10) Show that if we select any 14 integers from $S: \{1, 2, 3, \ldots, 25\}$

$\exists$ at least 2 whose sum is 26

11) let $X = \{n \in \mathbb{N} \mid 1 \leq n \leq 31\}$

let R be the relation on X defined as, $i R j$ if the ith & jth days of January 2327 will fall on the same day of the weeks

Show R is an equiv relation

12) Which of the following are equivalence relations (Prove or disprove)

(a) aRb if $a | b$

(b) $A \subseteq \mathbb{R}^2$ A set of all lines

$l_1, l_2 \in A \quad l_1 R l_2$ if l_1 perpendicular l_2

(c) xRy if $x+y$ odd $\quad x, y \in \mathbb{N}$