TABLE OF CONTENTS

WELCOME!

RUMBUS 2004 is the second annual "Research by Undergraduates in Mathematics Boston University Symposium", devoted to research by undergraduates, designed as a forum for students of the greater Boston area and beyond to come together and share their research in a poster session and short talks, in addition to learning from an outstanding keynote speaker. Panel discussions will be devoted to issues relevant to all mathematics students, in particular those exploring the methods, strategies and possibilities for a career of success in mathematics and its applications. Audiences at any mathematical level, from high school to University faculty, are expected to be able to enjoy the presentations and discussions: Mathematics is the Lingua Franca of the 21st Century!

RUMBUS's home: `http://math.bu.edu/people/ep/RUMBUS04`

RUMBUS's Founding Father: `http://math.bu.edu/people/ep/RUMBUS03`

RUMBUS04 is an MAA NSF-RUMC sponsored activity. Partial support under NSF Grant DMS-0241090 is gratefully acknowledged. As well, we are proud of and grateful for the support of Boston University's Student Activities Office granted to the BU Chapter of the Mathematical Association of America, via the Undergraduate Student Fee.

SCHEDULE

Saturday, April 24, 2004

Venue: Metcalf Center for Science and Engineering (SCI), 590 Commonwealth Avenue,

Lobby at Blandfort Street Entrance for Registration
Room SCI 103 for Breakfast, Tea and Poster Session
Room SCI 107 for Panel Discussion, Keynote Lecture and Student Papers Session

Registration and Breakfast: 10:30–10:55 am

Welcoming Remarks: 10:55 am
Professor Steven Rosenberg, Chairman, Department of Mathematics and Statistics, Boston University

Panel Discussion, RESEARCH IN MATHEMATICS: 11–12:30 pm

- What is it?
  Mathematics is almost inevitably becoming useless. By now we have machines that can perform calculations and draw curves... – M. Allègre, French Minister of National Education, Research and Technology (Interview, France-Soir, November 23, 1999)
- Is it for me?

Panelists: Robert L. Devaney (Boston University), David A. Ellwood, Academy Director (Clay Mathematics Institute), Jane M. Hawkins (University of North Carolina at Chapel Hill), Arthur C. Heinricher (Worcester Polytechnic Institute), Suzanne L. Weekes (Worcester Polytechnic Institute)
Moderator: Benedetto Scimemi (University of Padua)

Keynote Address: 2–3 pm
A Dynamical System in Phyllotaxis,
Pau Atela-Cortés of Smith College

Reception and Student Poster Session: 3-3:30 pm
Student Papers Session: 3:30 pm - 5:30 pm
Session Chair: Sarah L. Mabrouk, Professor, Framingham State College, and Chair, Northeastern Section of the Mathematical Association of America
Speakers:
Jessica Ong, Boston University "A Study of Knots and Links in K_6 and K_7"
Ivan Christov, Massachusetts Institute of Technology "A Wavelet Method for Image Anti-Aliasing"
Michael Burr, Tufts University "Simplicial Depth: An Improved Definition, Analysis, and Efficiency for the Finite Sample Case"
Jacob Fox, Massachusetts Institute of Technology "On Rado's Boundedness Conjecture"
Karola Meszaros, Massachusetts Institute of Technology "On Genus One Circle Trees"
Maksym Fedorchuk, Massachusetts Institute of Technology "Simplicial polytopes and Robbins' conjecture"
Daniel Keesing, Tufts University "A 3D Image Reconstruction Technique for Diffuse Optical Tomography"
Kaiwen Leong (Calvin), Boston University "A Correct Newton-Raphson and a Better Halley"
David K. Milovich, Massachusetts Institute of Technology "Branch Product Relations"
Tiffany Caufield, Boston University "A Study of the Relations of Multiple Zeta Values"
Karen A. Yeats, University of Waterloo and Boston University "Small Counterexamples to Wilkie's Identity"

KEYNOTE ADDRESS

A Dynamical System in Phyllotaxis

Pau Atela-Cortés

Smith College

Pine cones, sunflowers, artichokes, daisies... A great number of plants exhibit spirals that appear in two sets according to their orientation, clockwise and counterclockwise. The study of these arrangements is called Phyllotaxis. Most often, when counting the spirals in each set, we get a pair of consecutive Fibonacci numbers. (These are 1, 1, 2, 3, 5, 8, 13, 21,... each the sum of the previous two.) We will talk about a simple mathematical model of meristem growth based on Dynamical Systems theory, which might give us insights into this remarkable phenomenon observed for centuries.

Simplicial Depth: An Improved Definition, Analysis, and Efficiency for the Finite Sample Case

Michael Burr

Tufts University

Faculty Sponsor: Diane Souvaine

As proposed by Liu 1990 the simplicial depth of a point x with respect to a probability distribution F on R^d is the probability that x belongs to a random simplex in R^d. The simplicial depth of x with respect to a data set S in R^d is the fraction of the closed simplices given by $d+1$ of the data points containing the point x. We propose an alternative definition for simplicial depth which continues to remain valid over a continuous probability field, but also fixes some of the problems for the finite sample case, including those discussed by Zuo and Serfling 2000. Additionally, we discuss the effect of the revised definition on the efficiency of previously developed algorithms and prove tight bounds on the value of the simplicial depth based on the half-space depth.

A Study of the Relations of Multiple Zeta Values

Tiffany Caufield

Boston University

Faculty Sponsor: J. William Hoffman

It is possible to define a multiplication law on the vector space formed by taking the finite linear combinations of words formed on an alphabet of two letters, the shuffle product. There is another product, the stuffle product, which also gives relations among MZV. Using Mathematica, we have written a series of programs for implementing the shuffle and stuffle products as well as the xtaylor algorithm of Minh and Petitot. It is conjectured that all relations among MZV over the set of rational numbers are generated by these relations. Using our programs, we can investigate these shuffle and stuffle relations.

A Wavelet Method for Image Anti-Aliasing

Ivan Christov

Massachusetts Insitute of Technology

Faculty Sponsor: Gilbert Strang

The problem of aliasing in computer-generated images fundamentally arises from the fact that a perfectly straight, sloped, line cannot be drawn on a computer screen. Traditionally this problem has been approached by filtering the high frequencies of the image in the Fourier domain. Although the approach works well in practice, it is a bit general and can filter more than the aliasing effects. Using the discrete wavelet transform I localize the edges in the image (i.e. the high frequency oscillations) using the Mallat-Zhong algorithm. Then using the time-frequency localization property of wavelets I filter only along the edges of the image by reducing the modulus of the wavelet transform at those points bellow the edge detection threshold. Thus, effectively anti-aliasing the image.

Simplicial polytopes and Robbins' conjecture

Maksym Fedorchuk

Massachusetts Insitute of Technology

Faculty Sponsor: Igor Pak

We shall discuss the construction problem for convex polytopes. Given the combinatorial structure of a simplicial polytope, there is an open condition on the lengths of its edges, such that there exist a polytope with edges of given lengths. Aleksandrov's theorem gives sufficient conditions for such a construction. The problem of explicitly constructing a polytope in Euclidean space is important and very difficult. From a geometric point of view, it can be reduced to finding lengths of polytope's diagonals.

Algebraic relations between lengths of edges and lengths of diagonals (or any other metric invariant) of a simplicial polyhedron is the principal tool in our study. Their existence is also the main result of our paper. As an example of our approach, we prove the conjecture of Robbins on the degrees of generalized Heron polynomials for an inscribed n-gon.

To establish Robbins' conjecture we explicitly compute polynomials giving the algebraic relation between a radius of the inscribed n-gon and its sides.

This is joint work with Igor Pak.

On Rado's Boundedness Conjecture

Jacob Fox

Massachusetts Institute of Technology

Faculty Sponsor: Michael Artin, Daniel Kleitman and David Vogan

In this talk, we will discuss Rado's Boundedness Conjecture, an outstanding problem in Ramsey theory on the integers. It was conjectured by Richard Rado in his 1933 seminal dissertation "Studien zur Kombinatorik". We recently proved the first nontrivial case, which is stated below:

Rado's Boundedness Conjecture in 3 variables: If a, b, c are fixed rational numbers, and if every 6-coloring of the positive integers must have a monochromatic solution to $ax + by + cz = 0$, then every finite coloring of the positive integers must have a monochromatic solution to $ax + by + cz = 0$.

I will also show how this research leads to the proofs of several recent conjectures in literature. This talk will be accessible to a general audience. Joint work with Prof. Daniel Kleitman.

A 3D Image Reconstruction Technique for Diffuse Optical Tomography

Daniel Keesing

Tufts University

Faculty Sponsor: Misha Kilmer

Diffuse Optical Tomography (DOT) is a promising modality for imaging biological tissue. Two of its major uses are functional brain imaging and localization of tumors in breast tissue. This imaging technology is especially attractive because measurements are fast and non-invasive.

Our focus is on the development of iterative 3D image reconstruction algorithms. We have developed software that inputs a set of measured optical data to estimate the optical properties (absorption and scattering) within the tissue of interest. Contrast within these images could indicate a potential tumor in breast tissue or a region of activation in the brain, for example.

In particular, we have developed an ellipsoidal parametric representation for the anomaly characteristics. Instead of solving for the optical properties at every voxel (3D pixel) in the volume, we now only need to solve for a small number of parameters that define an ellipsoid. Doing so inherently eliminates the need for edge detection, and it could also improve the speed of the image reconstruction. The physics behind DOT will be briefly discussed, along with the forward model and the design of our algorithm.The motion of a fish's caudal fin is modeled as a flexible boundary

A Correct Newton-Raphson and a Better Halley

Kaiwen Leong

Boston University

Faculty Sponsor: Isaac Fried

Let $x = a$ be a fixed point of $x = f(x)$ and assume that $f(x)$ is differentiable to any degree at point a. The fundamental fixed point iteration theorem states that if $f'(a) = 0, f''(a) = 0, \ldots, f^m(a) = 0$ but $f^{m+1}(a) \neq 0$, then the the sequence $\{x_n\}$ produced by the iterative method $x_{n+1} = f(x_n)$ is such that $|a - x_{n+1}| < c|a - x_n|^{m+1}$ for some constant c. We propose to modify the fixed point iteration into the form $x_{n+1} = x_n + g(x_n)f(x_n) = F(x_n)$ and select $g(x)$ to assure that $F'(a) = 0, F''(a) = 0, F'''(a) = 0, \ldots, F^{(m)}(a) = 0$ so as to achieve high order convergence. Moreover, we propose to select specifically suitable functions $g(x)$ to accommodate specific $f(x)$ so as have, not only high order, but also efficient iterative methods for very accurate computations. The classical, quadratic, Newton-Raphson method is an immediate consequence of this fixed point iteration procedure. The cubic iterative methods proposed appear to be more efficient then those derived from the Halley method. Higher order iterative schemes are systematically generated for the efficient computation of some elementary functions and their inverses.

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On Genus One Circle Trees

Karola Meszaros

Massachusetts Institute of Technology

Faculty Sponsors: Richard P. Stanley

A genus one labeled circle tree is a tree with its vertices lying on a circle and edges drawn as straight lines within the circle, such that the circle together with the tree can be embedded in a surface of genus one, but not in a surface of genus zero. We introduce a novel reduction process on circle trees that we call e-reduction. By e-reduction a special type of subtree, an e-graph, is collapsed to an edge. We show that genus is invariant under e-reduction. Our main result is a classification of all genus one circle trees into nineteen classes. We obtain this classification by examining the genus one circle trees that are irreducible under e-reduction. Using our classification we prove a modified version of David Hough's conjecture, namely, that the number of genus one circle trees on n vertices is divisible by n, or if it is not divisible by n then it is divisible by $n/2$. Moreover, we explicitly characterize when each of these possibilities occur.

Branch Product Relations

David Milovich

Massachusetts Institute of Technology

Faculty Sponsor: Johnathan Farley

Given a set of sequences of integers, one may order the sequences lexicographically, that is, one may compare two sequences just by comparing them at the first place at which they differ. Branch product relations are a broad generalization of lexicographic orderings. Indeed, they are general enough to include direct products, lexicographic products, and ordered sums as special cases.

Given a well-founded poset X (i.e. a poset without infinite descending sequences), every bounded chain in X has a nonempty set of initial successors. If we give each such set the additional structure of a relation, say, a partial ordering, then the branch product of these relations is a certain induced relation on the set of maximal chains of X.

There are several nice results about branch product relations, including theorems on preservation of poset dimension and lattice completeness.

A study of knots and links in K_6 and K_7

Jessica Ong

Boston University

Faculty Sponsor: David Rohrlich

A one-to-one mapping of a graph in $\mathbb{R}^2$ into the three-dimensional space $\mathbb{R}^3$ is called a spatial embedding. Based on Conway and Gordon's work "Knots and Links in Spatial Graphs" (*J. Graph Theory* **7**, 1983, 445-453), this paper examines spatial embeddings of the complete graphs K_6 and K_7. Using properties of knots and links, we prove that there exists a certain invariant of the embedding for each type of graph. Then we find a particular example of a link in K_6 and a knot in K_7 and prove that all embeddings of such graphs carry such a feature, due to its invariance.

Small counterexamples to Wilkie's identity

Karen A. Yeats

University of Waterloo and Boston University

Faculty Sponsor: Stanley Burris

There are 11 obvious identities of the natural numbers with addition, multiplication, and exponentiation. In 1981 Wilkie provided the first identity which is not a consequence of the 11. Stan Burris and I have found what is presently the smallest known model of the 11 obvious identites in which Wilkie's identity fails. Our model has 12 elements.

PARTICIPANTS

F=FACULTY, O=OTHER, S=STUDENT

HS stands for High School

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For specific assistance, we mention:

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The Department of Mathematics and Statistics

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Finally, although the Keynote Speaker, the Panelists, the Speakers and the Poster Presenters are the ones who did the 'real' work and preparation, you too, Students, Faculty and Aficionados who form the audience, are the life and the heart of this meeting in particular, and of mathematics in general. We do hope you enjoy the day, and we hope to see you back! Contact us for anything we might do or share. With sincerest thanks,

– The organizers of RUMBUS 2004, Tiffany Caufield and Emma Previato