1. For each of the following PDEs, (1) classify it as dynamic or equilibrium, (2) classify it as linear homogenous, linear inhomogenous, or nonlinear, and (3) state its order and (spatial) dimension.

(a) $u_t = x^2\, u_{xx} + 2\, x\, u_x$
(b) $u_{xx} - u_{yy} = \sin u$
(c) $u_{xx} + 2\, y\, u_{yy} = 3$
(d) $u_t + u\, u_x = 3\, u$
(e) $e^y\, u_x = e^x\, u_y$
(f) $u_t = 5\, u_{xxx} + x^2\, u + x$

2. Consider the wave equation $u_{tt} = 4u_{xx}$. Verify that each of the following are solutions to it:

(a) $\cos(x - 2t)$
(b) $\exp(x + 2t)$
(c) $x^2 + 2xt + 4t^2$

3. State and justify three additional solutions to the wave equation $u_{tt} = 4u_{xx}$.

4. On either side of a biological cell membrane is an aqueous solution of ions. The ions' concentrations are usually different between the inside and outside of the cell, giving rise to an electrical potential and a membrane voltage, V. The membrane itself is impermeable. If the membrane has ion channels (pores created by transmembrane proteins), ions can cross the membrane and V is dynamic. The reversal potential of those ions, E, is the membrane potential that balances their concentration gradient so there is no net flux across the cell membrane. In a cylindrical portion of a cell with only (very many) passive channels, the membrane potential $V(x,t)$ obeys the linear cable equation

$$\lambda^2 V_{xx} = \tau V_t + V - E,$$

where λ and τ are space and time constants and $x \in \mathbb{R}$ is the linear position. (This assumes that V is radially symmetric on the cylinder). State and justify two solutions to this PDE.

5. Let L and M be linear partial differential operators. Prove that these are also linear operators:

(a) $L - M$
(b) fL, where f is an arbitrary function of the independent variables
(c) $9000\, L$
(d) $L \circ M$ ($\circ$ is composition; apply M and then apply L.)