1. Using characteristic coordinates, solve the initial value problem for the transport equation

$$\partial_t u(t,x) + 2\partial_x u(t,x) = 1,\ u(0,x) = e^{-x^2}$$

2. A sensor situated at position $x = 1$ in a canal of swiftly-moving water monitors the concentration of a pollutant $u(t,1)$ as a function of t ($t \geq 0$). The pollutant is transported along the canal with wave speed $c = 3$. We would like to know the initial concentration of the pollutant. At what locations x can you determine the initial concentration $u(0,x)$ from the sensor data at position $u(t,1)$?

3. Consider the homogenous wave equation in one spatial dimension.

(a) Does d'Alambert's factorization method work when the wave speed depends on the spatial position, $c = c(x)$? Explain why or why not.
(b) What about when the wave speed depends on time, $c = c(t)$? Explain why or why not.

4. Consider the periodically forced wave,

$$\partial_t^2 - c^2\partial_x^2 = \sin(\omega t)\sin(kx)$$

with real ω and k and with zero initial displacement and velocity. What are the resonant frequencies of u?

5. Given a smooth solution $u(t,x)$ to the (homogenous) wave equation, let

$$E = \frac{1}{2}\left(u_t^2 + c^2 u_x^2\right),\ P = u_t u_x$$

be the associated energy and momentum densities, respectively.
(a) Show that $P_t = E_x$ and that $E_t = c^2 P_x$.
(b) Show that E and P each also satisfy the wave equation.