1. Compute the Fourier series of the following functions, each defined on the interval $[-\pi, \pi]$.

(a) $\operatorname{sign} x = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$
(b) x^2
(c) $\sin(x)\cos(x)$
(d) $x \cos x$

2. Consider the complex and real Fourier series for $f(x)$:

$$f(x) \sim \sum_{k=-\infty}^{\infty} c_k\, e^{ikx}, \text{ and}$$

$$f(x) \sim \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(kx) + b_k \sin(kx),$$

with the coefficients defined as in lecture. Compute c_0 in terms of a_0. What does c_0 tell you about $f(x)$?

3. Let $f(x) : [-\pi, \pi] \to \mathbb{R}$ be an even function. Prove that the Fourier series of $f(x)$ is a pure cosine series.

4. Let $\mathcal{S}[f]$ and $\mathcal{S}[g]$ be the Fourier series for $f(x)$ and $g(x)$, respectively. For each statement below: true or false, and why?

(a) $\mathcal{S}[2f] = 2\mathcal{S}[f]$
(b) The coefficients in $\mathcal{S}[f+g]$ are given by adding the corresponding coefficients of $\mathcal{S}[f]$ and $\mathcal{S}[g]$.
(c) The coefficients in $\mathcal{S}[fg]$ are given by multiplying the corresponding coefficients of $\mathcal{S}[f]$ and $\mathcal{S}[g]$.
(d) The Fourier series for $f(2x)$ is obtained by directly replacing x by $2x$ in $\mathcal{S}[f]$.

5. Write out a proof of the pointwise convergence theorem for Fourier series, below. You may assume that the underlying function, f, is continuous. Your proof should be (at least) approximately at the level of formality of the convergence theorem as stated below. You may draw on any published source you wish (textbook or similar). Cite your source(s).

Theorem. Let $\tilde{f}(x)$ be a 2π-periodic C^1 function on the real line. The partial Fourier sums $\mathcal{S}_n[f] = \sum_{k=-n}^{n} c_k\, e^{ikx}$ converge pointwise to $\tilde{f}(x)$ as $n \to \infty$.