1. Consider the Hodgkin-Huxley model and its two-dimensional reduction from lecture 9/17.

(a) Write the reduced dynamics explicitly in terms of V and U.

(b) For a few values of the injected current I, simulate and compare the time series of the full Hodgkin-Huxley model and this two-dimensional reduction.

(c) Plot the nullclines of the reduced model (the curves obtained by setting either $dV/dt = 0$ or $dU/dt = 0$) at $I = 0$ and for increasing values of I.

(d) Plot $dV/dt - dU/dt$ as a function of V and U (a heat map or surface plot). Where is $dV/dt \gg dU/dt$, and vice versa? Based on this, roughly where should the integrate-and-fire approximation be better than the binary approximation, and vice versa?

(e) Compare the responses of V and m, in the full Hodgkin-Huxley model, to subthreshold steps in I. Does m appear much faster than V? How about for suprathreshold values of I?

2. Consider the two-dimensional Fitzhugh-Nagumo model,

$$\begin{aligned} \frac{dv}{dt} &= v(v-a)(1-v) - w + I \\ \frac{dw}{dt} &= \epsilon(v - bw). \end{aligned} \tag{1}$$

Unless otherwise stated, take $b = 6$, $a = 1/2$, $\epsilon = 1/10$.

(a) Plot the nullclines at $I = 0$ and increasing values of I. How do they compare to those of the two-dimensional reduced HH model from problem 1? Are the two models' nullclines affected by I in the same way?

(b) Compute and plot the fixed points as a function of I (one plot for v and one for w).

(c) Choose a value of I in the regime with three fixed points. Compute the Jacobian matrix at each fixed point and determine their linear stability.

(d) Consider the regime where the model has a single fixed point. Plot its Hopf bifurcation curve in the (a, I) plane for several (admissible) values of $b < b^*$. How does b modulate the Hopf curve?