GRS MA 769: Mathematical Neuroscience

Contact and logistics

Course description: Fundamental questions, models, and methods in mathematical and theoretical neuroscience. Biophysical and reduced single-neuron models, synaptic plasticity and learning, population density and mean field approaches for neuronal networks. Mathematical methods as needed, such as applied dynamical systems and stochastic processes.

The relation of neural network structure to neural computation is at the heart of modern systems neuroscience. Understanding these relations also promises to shed light on the computational capacity of modern deep neural networks—a topic of great current interest in machine learning. This course will focus on theoretical tools to understand structure-function relationships in neural network models, with a focus on biologically motivated models. We will introduce classic theoretical methods for deriving links between the structure of anatomical or functional networks and their activity, drawn especially from dynamical systems and statistical physics, and examine recent applications and extensions of these in neuroscience. We will also examine the converse linkage—how neural activity impacts biological network structure through synaptic plasticity to implement learning algorithms.

Topics

- 1. Single-neuron dynamics: Hodgkin-Huxley (HH) dynamics, model reduction of HH to Fitzhugh-Nagumo (FN) and integrate-and-fire (IF). Background: membrane voltage biophysics, phase portraits, some bifurcations. (∼2 weeks)
- 2. 1-d and binary single-neuron models: integrate-and-fire models, stochastic integrateand-fire models, perceptrons and their classification capacity. Background: Multivariate moment/cumulant generating functions, Gaussian integrals, replica trick. $(\sim2$ weeks)
- 3. Population dynamics: population density approach. Background: Stochastic process, master equation, Fokker-Planck equation. (∼2 weeks)
- 4. Population dynamics: mean field theory for randomly coupled networks. Background: Stochastic differential equations, path integral formulation. (∼3 weeks)
- 5. Long-term synaptic plasticity: Hebbian plasticity and PCA (∼1 week)
- 6. Long-term synaptic plasticity: Biophysical models and STDP (∼1 week)
- 7. Project presentations (∼2 weeks)

Student work and evaluation Grades will be based off of problem sets and literature assignments (50%) and a final project (50%).

Prerequisites The course assumes prior background at the undergraduate level in multivariable calculus, differential equations, linear algebra, and probability, or else permission of the instructor. We will balance further technical background with neuroscience topics as needed, possibly including elements of nonlinear dynamics, variational calculus, stochastic processes & stochastic differential equations, and perturbative and asymptotic expansions.

Texts In addition to classic and recent journal articles, other material may be drawn from texts including:

- Mathematical Foundations of Neuroscience; Ermentrout & Terman (ET).
- Statistical field theory for neural networks; Dahmen & Helias ($arXiv:1901.10416$)
- Principles of deep learning theory. Roberts, Yaida & Hanin. (arXiv: 2106.10165)
- Introduction to the theory of neural computation; Hertz, Krogh & Palmer, and/or Statistical mechanics of learning; Engel & van der Broeck
- Neuronal dynamics; Gerstner, Kistler, Naud & Paninski. (GKNP) (https://neuronaldynamics.epfl.ch)

Accessibility: I take the responsibility to foster an inclusive environment seriously. Please let me know if there are aspects of the course that impede your participation. In addition, the Office of Disability and Access Services can be contacted at 617-353-3658 or access@bu.edu.

Academic conduct: Boston University's academic conduct code can be found online at www.bu.edu/cas/current-students/phd-mfa-students/academic-policies-and-conduct-code/. All Boston University students are expected to maintain high standards of academic honesty and integrity. It is your responsibility to be familiar with the Academic Conduct Code, which describes the ethical standards to which BU students are expected to adhere and students' rights and responsibilities as members of BU's learning community.