1.7 Questions

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In a programmed learning environment, whether the interface is with a PC or with `Mathematica` notebooks or with a MAC, the student cannot ask questions. The give and take of questions and answers is a critical aspect of the human part of the teaching process. Teachers are *supposed* to answer questions.

There is more to this than meets the eye. When I say that a teacher answers questions I do not envision the student saying, "What is the area of a circle?" and the teacher saying " πr^2". I instead envision the student struggling to articulate some confusion and the experienced teacher turning this angst into a cogent question and then answering it. To do this well requires experience and practice. I frequently find myself responding to a student by saying, "let's set your question aside for a minute and consider the following." I then put the student at ease by quickly running through something that I know the student knows cold, and that serves as a setup for answering the original question. With the student on my side, I can answer the primary problem successfully. The point is that some questions are so ill-posed that they literally cannot be answered. It is the teacher's job to make the question an answerable one and then to answer it. See also Section 3.12 on asking and answering questions. *

A similar, but alternative, scenario is one in which the student asks a rather garbled question and I respond by saying, "Let me play the question back for you in my own words and then try to answer it ...." The point is that the responses "Your question makes no sense" or "I don't know what you mean" are both insulting and a cop-out. To be sure, it is the easy answer. But you will pay for it later. It takes some courage for the student to ask a question in class. By treating questions with respect, you are both acknowledging this fact and helping someone to learn. If instead you stare at the student as though he has weevils in his eyebrows, then you will gain no allies and will most likely lose several friends and make a few enemies.

Yet another encouraging response to a student question is to say, "Thank you. That question leads naturally to our next topic ..." Of course you must be quick on your feet in order to be able to pull this off. It is worth the trouble. Students respond well when they are treated as equals—see the discussion of this technique at the end of Section 1.1.

There are complex issues involved here. A teacher does not just lecture and answer questions. A good teacher helps students to discover the ideas. There are few things more stimulating and rewarding than a class in which the students are anticipating the ideas because of seeds that you have planted. The way that you construct your lecture and your course is one device for planting those seeds. The way that you answer questions is another.

When I discuss teaching with a colleague who has become thoroughly disenchanted with the process, I frequently hear complaints of the following sort: "Students these days are impossible. The questions that they pose are unanswerable. Suppose, for example, that I am doing a problem with three components. I end up writing certain fractions with the number 3 in the denominator. Some student will ask 'Do we always put a 3 in the denominator when doing a problem from this section?' How am I supposed to answer a question like that?" (See also Section 4.5 on frustration.)

Agreed, it is not obvious how to answer such a question, since the person asking it either **(i)** has not understood the discussion, **(ii)** has not been listening, or **(iii)** has no aptitude for the subject matter. It is tempting to vent your spleen against the student asking such a question. Do not do so. The student asking this question probably needs some real help with analytical thinking, and you cannot give the required private tutorial in the middle of a class hour. But you can provide guidance. Say something like "When a problem has three components it is logical that factors of 1/3 will come up. This can happen with certain problems in this section, or in any section. But it would be wrong to make generalizations and to say that this is what we do in all problems. If you would like to discuss this further, please see me after class." In a way, you are making the best of a bad situation. But at least you are doing something constructive, and providing an avenue for further help if the student needs it.

The Dalai Lama once visited the headquarters of *Time* magazine in Chicago. He was given the chef's tour, and then there was a grand formal lunch at which the various executives of the enterprise pontificated *ad nauseum.* The Dalai Lama—an elfin man—sat swathed in his saffron robe, an inscrutable smile on his face, saying nothing. After about an hour, the CEO of *Time* turned to the Dalai Lama and said, "Do you have any questions about *Time*, the nation's premiere news magazine? Go ahead, ask us anything at all." The Dalai Lama bowed his head for a moment, apparently deep in thought. Then he looked up and said, "Why do you publish it?"

We mathematicians are very much like the executives of *Time* in that story. We are wrapped up in our own world, we all speak the same language, and we suffer intruders with pained resignation. Sadly, our students are like intruders. They come to us with questions we would never have dreamed of, and they expect answers. They do not speak our language, and they do not necessarily

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respect our mores. Yet it is our job to talk to our students, to engage them in discourse, to answer their questions. We must exercise patience in order to gain their trust. And we must try to speak to them in their language, rather than in our own.

Let us consider some further illustrations of the principle of making a silk purse from a sow's ear—that is, answering unexpected or awkward questions in a constructive manner. The first example is a simple one.

> **Q:** Why isn't the product rule $(f \cdot g)' = f' \cdot g'$?

The answer is *not* "Here is the correct statement of the product rule and here is the proof." Consider instead how much more receptive students will be to this answer:

> **A:** Leibniz, one of the fathers of calculus, thought that this is what the product rule should be. He recorded this thought in his diary. Ten days later, he gave the correct form—with a proof and the cryptic statement that he had known this to be the correct form "for some time". Because we have the language of functions, we can see quickly that Leibniz's first idea for the product rule could not be correct. If we set $f(x) = x^2$ and $g(x) = x$ then we can see rather quickly that $(f \cdot g)'$ and $f' \cdot g'$ are unequal. So the simple answer to your question is that the product rule that you suggest gives the wrong answer. Instead, the rule $(f \cdot g)' = f' \cdot g + g' \cdot f$ gives the *right* answer and can be verified mathematically.

The second example is more subtle.

> **Q:** Why don't we divide vectors in three-space?

The *wrong* answer is to tell about Stiefel-Whitney classes and that the only Euclidean spaces with a division ring structure are $\mathbb{R}^1, \mathbb{R}^2, \mathbb{R}^4$, and $\mathbb{R}^8$. A better answer is as follows.

> **A:** J. Willard Gibbs invented vectors to model physical forces. There is no sensible physical interpretation of "division" of physical forces. The nearest thing would be the operations of projection and cross product, which we will learn about later.

Notice that in both illustrations an attempt is made to turn the question into more than what it is—to make the questioner feel that he has made a contribution to the discussion. *

> **Q:** Why isn't the concept of velocity in two and three dimensions a number, just like it is in one dimension?

If you are in a bad mood, you will be tempted to think that this person has been dreaming for the past hour (or the past week!) and has understood absolutely nothing that you have been saying. Bear up. Resist the temptation to voice your frustrations. Instead try this:

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> **A:** Let me rephrase your question. Instead let's ask, "Why don't we use vectors in one dimension to represent velocity just as we do in two and three dimensions?" One of the most important features of vector language is that a vector has *direction* as well as magnitude. In one dimension there are only two directions—right and left. We can represent those two directions rather easily with either a plus sign or a minus sign. Thus positive velocity represents motion from left to right and negative velocity represents motion from right to left. The vector language is *implicit* in the way that we do calculus in one dimension, but we need not articulate it because positivity and negativity are adequate to express the directions of motion.
>
> In dimensions two and higher there are infinitely many different directions and we therefore require the explicit use of vectors to express velocity.

As the author of this book, I have the luxury of being able to sit back and drink coffee and think carefully about how to formulate these "ideal" answers to poor questions. When you are actually teaching you must be able to do this on your feet, either during your office hour or in front of a class. At first you will not be so articulate. This is an acquired skill. But it is one *worth acquiring*. It is a device for showing respect for your audience, and in turn winning *its* respect.

Large lectures pose special problems with the issue of student questions. Obviously you cannot let each student ask his little question. You cannot let your lecture get bogged down with questions like "How do you do problem 6?" or "Will this be on the test?" See Section 2.14 for a discursive discussion of questions in the large lecture context.

A final note about questions. Even though you are an authority in your field, there are certainly things that you do not know. Occasionally these lacunae in your knowledge will be showcased by a question asked in class or during your office hour (it does not happen often, so don't get chills). The sure and important attribute of an intelligent, educated individual is an ability to say, "I don't know the answer to that question. Let me think about it and tell you next time." On the (rare) occasions when you have to say this, be sure to follow through. If the item that you don't know is an integral part of the class—and this had better not be the case very often—get it down cold because the question is liable to come up again in a different guise later in the course. If it is not an integral part of the course, then you have no reason to feel bad. Just get it straight and report back.

The main point is that you should never, under any circumstances, try to fake it. If you do, then you will look bad, your interlocutor will be frustrated and annoyed, and you will have served no good purpose. If there is any circumstance in which honesty is the best policy, this is it. Professor of Economic History Jonathan R. T. Hughes was wise to observe that "There is no substitute for knowing what you are talking about."

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3.8 Inductive vs. Deductive Method

It is of paramount importance, epistemologically speaking, for us as scholars to know that mathematics can be developed *deductively* from certain axioms. The axiomatic method of Euclid and Occam's Razor has been the blueprint for the foundations of our subject. Russell and Whitehead's *Principia Mathematica* is a milestone in human thought, although one that is perhaps best left unread. Hilbert and Bourbaki, among others, also helped to lay the foundations that assure us that what we do is (for the most part) logically consistent.

However mathematics, as well as most other subjects, is not learned deductively; it is learned *inductively*. We learn by beginning with simple examples and working from them to general principles. Even when you give a colloquium lecture to seasoned mathematicians, you should motivate your ideas with good examples. The principle applies even more assuredly to classes of freshmen and sophomores.

Beresford Parlett recently said

> Only wimps do the general case. Real teachers tackle examples.

*

This simple idea should be a guiding force whenever you are preparing to explain a new idea to your students.

Take the fact that the mixed partial derivatives of a C^2 function in the plane commute. To state this theorem cold and prove it—before an audience of freshmen—is showing a complete lack of sensitivity to your listeners. Instead, you should work a couple of examples and then say, "Notice that it does not seem to matter in which order we calculate the derivatives. In fact there is a general principle at work here." Then you state the theorem.

Whether you actually give a proof is a matter of personal taste. With freshmen I would not. I'd tell them that when they take a course in real analysis they can worry about niceties like this. Other math instructors may have differing views about the question of proof.

And by the way—you know and I know that C^2 is too strong a hypothesis for the commutation of derivatives. But, really, isn't that good enough for freshmen? If a bright student raises this issue, offer to explain it after class. But do not fall into the trap of always stating the sharpest form of any given result. Great simplifications can result from the introduction of slightly stronger hypotheses, and you will reach a much broader cross section of your audience by using this device.

*

Ralph Boas had these thoughts about the inductive method:[4]

> I once heard Wiener admit that, although he had used the ergodic theorem, he had never gone through a proof of it. Later, of course, he did prove (and improve) it.
>
> ......
>
> I do not think my story about Wiener is very surprising. One can't always be going back to first principles.

[4] Part of this quotation comes from a private communication between Boas and James Cargal, as quoted in [CAR].

> I quite agree that—at least for *some* people (I am one of them) calculation precedes understanding. I have probably said before that I knew how to calculate with logarithms long before I knew how they worked. The idea that proofs come first is, I think, a modern fallacy. Certainly—even in this calculator age—a child learns that $2 \times 2 = 4$ before understanding *why*. The trouble with "new math" was (in part) the fallacy of thinking that understanding needs to come first.

Ralph Boas was a great teacher, and there is wisdom in what he says. Don't put the cart before the horse when you teach. A young student is ill-motivated to learn the inner workings of a mathematical idea before he has understood what it is and how to use it.

Now suppose that you are teaching real analysis (from [RUD], for example). One of the neat results in such a course is that a conditionally convergent series can be rearranged to sum to any (extended) real limit. When I present this result, I first consider the series $\sum(-1)^j/j$ and run through the proof specifically for this example. The point is that, by specializing down to an example, I don't have to worry about proving first that the sum of the positive terms diverges and the sum of the negative terms diverges. That is self-evident in the example. Thus, on the first pass, I can concentrate on the main point of the proof and finesse the details. After doing the first example, and thereby instilling the students with confidence and understanding, I easily can go back to the general result and describe quickly how it works.

Go from the simple to the complex—not the other way. It's an obvious point, but it works. An example of this philosophy comes from the calculus. Many calculus books, when they formulate Green's theorem, go to great pains to introduce the notions of "x- simple domain" and "y- simple domain" (i.e. domains with either connected horizontal or connected vertical cross sections). This is because the authors are looking ahead to the proof, and want to state the theorem in precisely the form in which it will be proved. The entire approach is silly.

Why not state Green's theorem in complete generality? Then it is simple, sweet, and students can see what the principal idea is:

Theorem: *Let $\Omega \subseteq \mathbb{R}^2$ be a smoothly bounded domain. Let $\mathbf{F}(x,y) = u(x,y)\mathbf{i} + v(x,y)\mathbf{j}$ be a smooth vector field defined on Ω together with its boundary. Then*

$$\oint_{\partial\Omega} \mathbf{F} \cdot d\mathbf{r} = \iint_{\Omega} \operatorname{curl} \mathbf{F}(x,y)\, dxdy.$$

When it is time for the proof, just say "to keep the proof simple, and to avoid technical details, we restrict attention to a special class of domains ..." This approach communicates exactly the points that you wish to convey, but cuts directly to the key ideas and will reach more of the students with less fuss. If a student asks you afterward what is lost by restricting attention to x-simple and y-simple domains, you can point out that more general (smoothly bounded) domains can be cut up into x-simple and y-simple domains. This is a *wonderful*

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answer, for the apt student will immediately begin to draw pictures and cut up domains, and will gain immense satisfaction from the process.

Here is a useful device—almost never seen in texts or discussed in teaching guides—that was suggested to me by Paul Halmos:

Suppose that you are teaching the fundamental theorem of algebra. It's a simple theorem. You could just state it cold and let the students think about it. But the point is that these are *students*, *not* mathematicians. It is your job to give them some help and motivation. First present to them the polynomial equation $x - 7 = 0$. Point out that it is easy to find all the roots and to say what they are. Next treat $2x - 7 = 0$. Follow this by $x^2 + 2x - 7$ (complete the square—imitating the proof of the quadratic formula). Give an argument that $x^3 + x^2 + 2x - 7$ has a root by using the intermediate value property. Work a little harder to prove that $x^4 + x^3 + x^2 + 2x - 7$ has a root. Then surprise them with the assertion that there is no formula, using only elementary algebraic operations, for solving polynomial equations of degree 5 or greater. (In the process, tell the students a little bit about Evariste Galois, and how he recorded his key ideas the night before the duel in which he was sure he would die—at the age of 21!) Finally, point out that the remarkable fundamental theorem of algebra, due to Gauss, guarantees in complete generality that any non-constant polynomial has a (complex) root.[5]

Notice how much depth and texture this simple discussion lends to the fundamental theorem. You have really given the students something to think about. Stating the theorem cold and then moving ahead, while *prima facie* logical and adequate, does not constitute teaching—that is, it does not contribute to understanding. As with many of the devices presented in the present book, this one becomes natural after some practice and experience. At the beginning it will require some effort. The easiest thing in the world for a mathematician to do is to state theorems and to prove them. It requires more effort to *teach*. *

I once heard a splendid lecture by a distinguished mathematical physicist. He began by telling us that everyone thinks that someone else understands the Second Law of Thermodynamics. But in fact nobody really understands it. He went on to say that he had come up with a new formulation of the Second Law, and he could now say that he understood at least *this new formulation*. The rest of the talk concentrated on explaining his new idea. He began that portion of the discussion by saying, "Suppose you have a cup of coffee ..." Throughout the talk, he illustrated sophisticated ideas from mathematical physics by way of the cup of coffee.

Think a minute about what this speaker did for his audience. First, he exhibited incredible humility by saying that, up until this moment, he had never understood the Second Law of Thermodynamics. Then he went on to say that he found a new way to think about it that at least *he* understood.[6] To illustrate his new ideas, he spoke of a cup of coffee. Who in the audience would not be at ease after an introduction like this? Who would not be dying to hear more? Who would not feel that the speaker was welcoming him into his world? Isn't this what you want to do when you teach?

One could go on at length about the philosophy being promulgated here. But the point has been made. Saki once said that, "A small inaccuracy can save hours of explanation." Mathematicians cannot afford to be inaccurate. But, for the students' sake, we can simplify. We can reach out to our audience, and find a meeting ground. We can communicate.

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of exceptions that are connected with dealing with *people*. I have used the "incomplete" here as but one example of the problems and potential enigmas that can arise. Your department probably has set policies, or at least guidelines, for handling incompletes. Become acquainted with the routine procedures before you give your first "I".

4.5 Frustration

One of the most commonly heard complaints of college mathematics instructors, especially experienced instructors, is this: "Math 297 is a prerequisite for the course that I am teaching yet the students don't seem to know anything from Math 297." A variant of this is "My calculus students cannot[1] add fractions" or "My calculus students don't know how to expand $(a + b)^2$.".

Indeed, these are valid complaints. It is also valid to complain about the high cost of living, or about death and taxes. The peccadillos described in the first paragraph are facts of life and we as math instructors must deal with them. The truth is that we instructors think about math all day long, every day. We see the entire curriculum as a piece. For the experienced math instructor, there are no seams and creases between linear algebra, calculus, differential equations, and so forth. We swim effortlessly through the ideas, using whatever tools are needed. (By the way, if this doesn't exactly describe you then don't panic—I'm using a bit of poetic license here.) Students are different. They think about math when they are in the math classroom and (one hopes) for a few designated hours outside the classroom, but they are not inured in the subject.

So what is the point? It is simple. If you are in the second week of freshman calculus and you need to add two algebraic expressions that are fractions, then gently remind the students how to do it. If you need to expand the expression $(a + b)^2$, then say, "You remember how this works—right?" After a few gentle reminders, most of the students fall into the flow and they *will* remember how it goes.

Take a break and watch the "Tonight Show" or "Late Night". Listen to the monologue. If the host is going to crack a joke about someone slightly less famous than Bill Clinton, then he gently reminds the audience who it is that he is talking about. It's just good sense. These television hosts can be even less sure of how well informed their audiences are than we can be in our math classes. They guarantee that their viewers will *understand* by providing a bridge.

Contrast these recommendations with the following rather common alternative. The professor needs to add two fractional algebraic expressions, so he just barrels through it (rapidly and without comment, as one would do for a colleague). After a few moments some hands are raised, some hesitant questions are asked, and it soon becomes clear that many students are lost. The professor says, "What is the matter with you people? This is high school stuff. Am I a

[1] The Bank of America in Westwood Village, Los Angeles used to regularly place an advertisement in the UCLA student paper each spring. The purpose of the ad was to encourage members of the latest graduating class to consider a career with B of A. The ad read in part "applicants must be able to add fractions." So it is not just math teachers who are plagued by this problem.

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baby sitter or what?" (I'm not making this up; I have colleagues who do just this.)

In my view the behavior of the mathematics instructor in the last paragraph is the mathematical analogue of shooting one's self in the foot. The professor (perhaps unconsciously) *sets up* a situation for failure. And there is no good or useful point to it. It not only sets a bad tone for that day, but also for the remainder of the course. The instructor needs only to expend just a little extra effort to anticipate these pitfalls, and to devote a few seconds to allaying them. And it does a world of good.

In fact, for me, helping the students recall how to add fractions or to expand quadratic expressions (or any other analogous elementary operation) is a form of protective coloration. It is too easy for me to make errors when doing these elementary operations. If I whip through them, leaving most of the students in the dust, then I in fact increase the likelihood that I will make an error, and also abrogate any sympathy I might have garnered when my error was detected. If instead I slow down and walk the students through the calculation, then it becomes "our calculation". They help me to check it, and the chance of an error is reduced to virtually nil.

As indicated at the beginning of this section, these frustrations also present themselves at more advanced levels—even with math majors. As an instance, linear algebra is often a prerequisite for multi-variable calculus. And well it should be, for matrix language is a natural vehicle for expressing the derivative, the chain rule, and so forth. But it is an artifact of the American mathematics curriculum that linear algebra is often taught in a vacuum. The students have no hooks to hang the ideas on, and they do not remember them very well. There is no alternative, if you want to keep your multi-variable calculus course on an even keel, to giving a whirlwind review of the salient linear algebra ideas as you use them. Here, by "whirlwind", I mean a five or ten minute snapshot, on the fly, of the relevant idea right before it is used.

Here is another way to look at the matter of frustration. You and I have become accustomed, when we visit our physician or our attorney or our psychotherapist, to a certain amount of professional decorum. Often the doctor or lawyer or counselor meets us in a well-appointed office, dressed in a suit or other formal attire, and he exudes courtesy, detachment, and professionalism. Sadly, academics do not seem to have bought into this game. If you had a fight this morning with your spouse, or got a speeding ticket when driving to work, or got a rejection of your latest paper in the mail, then you are liable to take it out on your students. Your office may look like a pigsty and your haberdashery like something from Zola's *La Terre*. But you have set a standard for your students and if they don't meet it then you may lose your objectivity or your patience and you may react.

If you have been dutifully teaching your calculus students for several weeks running, and if their latest midterm shows that they've absorbed very little of your wisdom, then you are liable to vent your spleen at them. You would never expect your family doctor to start hollering at you about losing weight, nor your lawyer to scream at you about paying your taxes on time, nor your psychiatrist to excoriate your for being too neurotic. But you and I sometimes find ourselves

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altogether losing it and—more is the pity—giving hell to our students.

Of course there are notable ways in which doctors and lawyers differ from academics. What makes us special is that we endeavor to impart knowledge to our students *and we expect them to radiate it back at us.* When they fail to do so, then we are disappointed, sometimes angry, and certainly frustrated. What I am suggesting here is that it can serve to your advantage to set yourself apart from your students. Maintain some objectivity. Try not to become emotionally involved. As award-winning teacher David S. Moore observes (see [MOO] as well as Section 3.1), teaching is a job: You prepare your class and you go do it. If there are problems, you deal with them. If the students aren't learning then you teach harder. It is part of the academic milieu, and part of our training, to think of ourselves like operatic divas: If things don't go as they should then perhaps a tantrum is in order. Not so. Be strong. *

If the students are not working hard enough, nor absorbing the ideas at a pace and depth to suit your ideals, then too bad. But it's too bad for them; it's really no big thing for you. Teaching freshman is like mowing your lawn. No matter how good a job you do this Saturday, you are going to have to do it again next Saturday. Yelling at the lawn doesn't help.

Of course I am disappointed when my students—despite my best efforts—can't do three-dimensional graphing, or can't understand Stokes's theorem, or can't apply the ϵ-δ definition of continuity. But my job is to teach and I just get in there and do it. If I have to cover a tricky topic twice, or even three times, then that's the breaks. Part of being a successful teacher is gaining your students' trust. Go watch the movie *Stand and Deliver* about the legendary calculus teacher Jaime Escalante. He was tough on his students. He told them when he was disappointed and he worked them hard. But he never belittled them, and he never lashed out at them. He showed genuine pride and enthusiasm when they did well. The most important thing he did for his students is that he made them believe in him. They worked hard for *him* because they trusted *him*.

The frustration problem described here is one of the few in this book that plague the experienced instructor somewhat more than the novice. Novices are usually drunk with youthful enthusiasm for teaching. Middle aged folks like myself are often just tired. We tend to lose our patience, and to forget the struggles of the uninitiated. An instructor who has been dealing with, and teaching, the ideas for twenty years cannot understand why students don't remember what they have already seen once. *Once*! The key to success here is to try to develop (or remember) a little sensitivity to the point of view of the students.

As a closing thought, consider the following. If your students are not speaking to you then it is probably because you are not speaking to them. You may be lecturing *at* them, you may be exhorting them, you may be talking down to them, or you may be venting your spleen and verbally abusing them. But you are not relating to them as people. You are not teaching. Try it—you'll like it.

4.6 Annoying Questions

At several junctures in this book I have mentioned some spine-tingling, bone-chilling, conversation-stopping questions that students can and will ask. One of these is, "What is all this stuff good for?" Another is, "Will this be on the test?" Another is, "Why don't you prepare your lectures more carefully? You are wasting our time."

If you are asked the third question then the fault probably lies with you. You should have done a better job preparing your class. If things are really going dreadfully, you might say to the class, "I apologize. This class is going very badly. Let's quit for today." Nobody will take this amiss, and it is probably the most diplomatic way out of an uncomfortable situation—but do not use this device more often than about once per semester. The best policy is to use forethought to prevent such an encounter.

Dealing with the first two questions, and others like them, is something that you will learn to do through bitter experience. In America in the 1990s, we endeavor to educate a broad cross section of the population. We cannot assume, as perhaps a don at Oxford could one hundred years ago, that our students are at the university primarily to learn to become refined citizens—and that they are happy to consider whatever we set before them. In particular, today's students are prone to *challenge* what we are doing. It is a part of your job to be prepared to answer their challenges. The challenges are not generally hostile, but having respect for your audience requires that you be prepared to provide a thoughtful response. If you accept my premise—repeated throughout this book, but particularly in Section 3.14—that getting an education is learning the art of discourse, then you should set an example for your students. If one of them poses an intelligent, well-thought-out question (even though it may be one that you don't particularly want to hear), then you should endeavor to answer it in a manner that is both correct and intellectually stimulating. Now let me say a few words about the particular queries pinpointed at the outset of this section.

Let us consider the question, "Will this be on the test?" One option that you have for an answer is the obvious one. Tell them that it *will* be on the test and then, indeed, put it on the test.

Now let us look at the opposite situation. If you are going to present something to your students and have no intention of testing them on it, then you have two choices. You can tell them up front that they will not be tested on it—they should just sit back and listen. Or you can tell them that they will be tested on it and then *don't* test them on it. Know consciously which choice you have made before you proceed.

If you choose the first route indicated in the last paragraph, then you might put the exercise in context like this: Explain that some ideas are difficult and deep. It requires several exposures to such an idea before it begins to make sense. This is an opportunity for the student to begin to ponder something important. Students are pleased to be treated like fellow scholars, and will usually act accordingly.

If you choose to tell the students that X will be on the test but in fact you have no intention of putting X on the test ... That is OK, but don't over-use

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this privilege. It will irritate the students and could damage your credibility.

The question "What is all this stuff good for?" is treated in Sections 1.7, 1.11. You, the mathematician, can get so wrapped up in your mathematics that such a question can catch you entirely off guard. Spend a few moments arming yourself against it.

The main point is this. You are not lecturing fellow mathematicians, who are inured to your point of view. You are lecturing students. Students will challenge you and ask questions. Some of these questions are difficult. If you want to retain the students' respect, then you must be prepared to deal with their queries and to understand their point of view.

4.7 Discipline (excerpt) 4.7

Not long ago I had a student come to me and tell me that he'd skipped the previous two weeks of class. But he was now returning and would do his best to catch up. I said "fine"—if he needed some guidance, then he should let me know. Indeed, he showed up in class that day. I began the lecture by saying, "OK, let's have another look at Stokes's theorem." My prodigal friend, who had just been in to see me, said, "Could you give a quick review of this concept?" (I had been discussing Stokes's theorem for most of his two week absence.) I am ashamed to say that I lost it. I said, "No. Not for someone who hasn't been to class for two weeks." The other students supported me in this. I could tell by looking in their eyes. But I felt like a rat. When I conducted my personal debriefing after class, I wondered whether I had done the right thing. It all came out well, because a few minutes later he came to my office and apologized to me, I apologized to him, and everything was hunky dory. In retrospect, I think I should have said, "We've been studying Stokes's theorem for two weeks, and it doesn't lend itself to a quick summary. See me after class if you want more help."

Always remember that you have the power to command respect, but you cannot *demand* it. If you present the image of an organized, knowledgeable scholar who is trying to do a good job of teaching, then most students will play ball with you. If instead you are a bumbling, unprepared clod who clearly doesn't care a damn for the class, then you can expect like treatment from the students.

4.8 Mistakes in Class

The most important rule to follow before giving a class is to prepare (Section 1.3). How much you prepare will depend on you—on your experience, your confidence, your training, and so forth. Being fully prepared gives you the flexibility to deal creatively with the unexpected.

But nobody is perfect. No matter how well prepared you are, or how careful, you will occasionally slip up. In the middle of a calculation, a plus sign can become a minus sign. An x may become a y. You will say one thing, think a second, write a third, and mean a fourth. It is best if you can handle these slips with a flair, and particularly without sending the class into a tailspin.

I endeavor in my classes to create an atmosphere in which students are comfortable to shout out, "Hey, Krantz, you forgot a minus sign." Or, "Is that a capital F or a lower case f?" This is a form of participation, and it can be a very constructive one. If you handle these situations badly, then students will be less inclined to ask questions or to approach you on other, more important, matters.

If mistakes are small, and occur in isolation, then they will not damage the learning process. But if they are frequent or, worse, if they snowball, then you will lose almost everyone, give a strong impression of carelessness, set a bad example, and (to oversimplify) turn off the class.

You may endeavor to bail out of an example that you are lousing up by saying, "Well, this isn't working out. Let's start another example." It won't work. This is in the vein of two 'wrongs' not making a 'right'. The only solution here is not to make mistakes and to handle those that you make anyway with a certain amount of finesse.

However: If you can see that the example you are working on is getting out of control, if you *know* that it is going from bad to worse, that you are so bolloxed up that you will be unable to bail out of it, then what do you do? Do not spend the rest of the hour trying to slug it out. Doing so is uncomfortable, counterproductive, and will not teach anyone anything. Instead apologize, say that you will write up the solution and hand it out next time (or put it on the class Web page!), and move on. My advice here may seem to fly in the face of Section 2.8, and to contradict the last paragraph, but it is only meant for extreme situations. Making mistakes is one of the surest ways to lose control of a class. It is the mathematical analogue of an equestrian letting go of the reins. Strive not to do it.

Besides preparing well, there are technical devices for minimizing the number of errors that you make. When I am working an example in a lower-division class, I pause *frequently* to say, "Let's make sure this is right" or, "Let's double check this step." I often pick out a student (who I know will respond well) and ask him whether that last step was done correctly. This procedure provides a good paradigm for the students. It also allows note takers to catch up and allows the bright students to strut their stuff in a harmless manner.

One of the most common ways that students make mistakes in their work is by trying to do too much in their heads. Therefore you should set a good example. Write out all calculations. Point out *explicitly* that you have had many years of experience with this material yet you still use lots of parentheses and write out every step.