

Why (and How) I Teach without Long Lectures

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1. Why I Gave Up Long Lectures

I could have benefited greatly from Steven Krantz's tips in 1962 when I taught my first class. In fact, I can see that over the years my lecturing style and techniques evolved to be remarkably similar to those Steven Krantz (SK) suggests. I was a very popular lecturer and recently won an MAA sectional award for distinguished teaching based in no small part on the lecture courses I gave at Illinois between 1968 and 1988. But for the last ten years, I have completely abandoned the long lecture method.

My last lecture effort was calculus in 1988. I thought I did a bang-up job, but the students did not respond with work anywhere near the level I was used to and have become used to after I gave up on introductory lectures—despite the fact that I had been giving the lectures largely in harmony with SK's recommendations.

Simply put, today's students do not get much out of long lectures, no matter how well they are constructed. The material comes too fast and does not sink in well. The students of the past responded by becoming quiet scribes. Today's students demand more action and accountability. That's why many students cut class and even when they come they often ask hostile questions such as "What's this stuff good for?" They do not read their texts. Some students even disrupt lectures. And as SK notes, many professors ask the questions

- Why won't my students talk to me?
- Why is class attendance so poor?
- Why won't students do their homework?
- Why do they perform so poorly on exams?

And then they shrug it off saying to themselves: "If only I had taught at Harvard things would be different. I would have bright and eager students." or "Students these days are impossible."

It is the lecture method of teaching that is impossible—the method of teaching via long lectures is crumbling under its own weight. This is true not just in mathematics. Across the University of Illinois, there is a major controversy about whether professional note takers may take notes and sell them to students who would rather not attend lectures. One of the first to note that the lecture system needed to be replaced was Ralph Boas in 1980: "As a means of instruction, lectures ought to have become obsolete when the printing press was invented. We had a second chance when the Xerox machine was invented, but we muffed it." Many math instructors are trying to teach today's students using only yesterdays tools and approaches. And neither the instructors nor the students pleased with the results.

Introductory lectures are not (and probably never have been) a particularly effective vehicle for introducing students to new material. A few strategically timed and strategically placed short follow-up lectures (sound bites) can be very effective. But the problem with introductory lectures is that they are full of words that have not yet taken on meaning and full of answers to questions not yet asked by the students. A further problem is that many lecturers fall into the trap of believing that their job is to think for the students. This effectively shunts the students to the sidelines—making them into mere scribes who verify in the homework and tests the math truths promulgated by the lecturer. As Bill Thurston put it: “We go through the motions of saying for the record what the students ‘ought’ to learn while students grapple with the more fundamental issues of learning our language and guessing at our mental models. Books compensate by giving samples of how to solve every type of homework problem. Professors compensate by giving homework and tests that are much easier than the material ‘covered’ in the course, and then grading the homework and tests on a scale that requires little understanding. We assume the problem is with students rather than communication: that the students either don’t have what it takes, or else just don’t care. Outsiders are amazed at this phenomenon, but within the mathematical community, we dismiss it with shrugs.”

In summary, I do not disagree with SK’s approach to lectures, as he gives some great advice, which I used to follow as well. However, I do question the necessity, importance, and educational quality of lectures as a method for students to learn mathematics.

2. What I Replaced Lectures With

Another piece of wisdom from Ralph Boas : “Suppose you want to teach the ‘cat’ concept to a very young child. Do you explain that a cat is a relatively small, primarily carnivorous mammal with retractile claws, a distinctive sonic output, etc.: I’ll bet not. You probably show the the kid a lot of different cats saying ‘kitty’ each time until it gets the idea. To put it more generally, generalizations are best made by abstraction from experience.”

Today my calculus, differential equations and linear algebra students get the experience they need through Mathematica-based courseware written by Bill Davis, Horacio Porta and me. The basic ideas are laid out in interactive Mathematica Notebooks in which new issues arise visually through interactive computer graphics. With this courseware, limitless examples are possible almost instantly. If the student doesn’t get the point right away, then the student can rerun with a new example of the student’s own choosing. They can use the courseware to touch and see the math “kitty” as many times as they want to. They see for themselves what the issues are before the words go on and generalizations are made. One of our favorite techniques is to give a revealing plot and ask the students to write up a description of what they are seeing and to explain why they see it. In these courses, conceptual questions are the rule and students answer them. Contrast this with the typical student problems assigned in traditionally taught mathematics courses.

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Here is the story behind the evolution of our courseware and the way it is used: In 1988-90, when Horacio Porta, Bill Davis and I were developing the original version of the computer-based course *Calculus&Mathematica*, Porta and I offered regular introductory lectures at Illinois. We noticed poor attendance and asked the students why. The students uniformly replied: "We don't need them. We can get what we need from the computer courseware when we need it. What we do want is a followup discussion from time to time." We followed their advice and have never seen the need to go back. Our students taught us how to teach. Over the years, almost all teaching of *Calculus&Mathematica* (and sister courses *DiffEq&Mathematica* and *Matrices, Geometry&Mathematica*) has evolved to this model (sometimes known as Studio learning, a term coined by Joe Ecker for his Maple-based calculus course): All the student problems are freshly written with the idea of engaging the student's interest. Assignments are made on Thursday. Students work on each assignment for one week. One day before the assignments is due, a classroom session is held to discuss what the week's work was all about. Students come armed with questions and if they don't fill up the whole hour, then the instructor gives a several pointed mini-lecture addressing points the students should have picked up during the last week. All other class meetings are in the computer lab with the instructor answering student questions as they arise—at the ultimate teachable moment. This lab interaction between teacher and student (which is sometimes done via e-mail) is very important. No longer are the students the professor's audience; students are the professor's apprentices.

The students' weekly assignments count for at least half their semester grades. Because there are no other lectures, the whole course consists of student work. In this model, it is what the students do that is important rather than what the professor says and how professor says it. Still the influence of the instructor is pervasive and the course ends up satisfying Gary Jensen's and Meyer Jerison's criteria: Setting pace, teaching students to read, and fully engaging the student in the learning process.

This learning model cannot be accomplished with traditional printed texts and traditional lectures and lends itself rather well (but not perfectly) to Internet distance education. NetMath centers offering via the Internet calculus, differential equations and linear algebra courses for university credit supported by live mentors have formed at several universities and colleges. Here is a reaction from a high school teacher who sponsors NetMath *Calculus&Mathematica* in Alaska: "Jessica has really enjoyed the course, and her father, a veteran of traditional calculus courses, is very impressed with the understanding of the mathematics that this method imparts. He has done all the problems and loved it. There have been some loud arguments—most of which she has won."

3. Content Issues

The trouble with the lecture system is compounded by the fact that our undergraduate courses, for the most part, have been frozen in the past and have become unable to adjust to modern demands. Undergraduate mathemat-

ics courses today are nearly indistinguishable from the undergraduate courses I took in 1960. Peter Lax put it this way in 1988: "The syllabus has remained stationary, and modern points of view, especially those having to do with the roles of applications and computing are poorly represented ..." When I look over mathematics undergraduate courses during this century, I see a smooth evolution of new ideas and better mathematics through the period 1900–1960. Topics of limited interest such as haversines, common logarithms, Hoerner's method, latus recta, involutes, evolutes, Descartes's rule of signs all had their time in the sun but were de-emphasized in favor of more important topics. And then the content became frozen. There is a whole list of 20th Century topics that have been by and large rejected in today's mathematics classroom. A short list: The error function, singular value decomposition for matrices, unit step functions and their "derivatives", the Dirac delta functions in differential equations, using the computer to plot numerically solutions of differential equations, Fast Fourier Transforms, wavelets. There is plenty of what Peter Lax calls "inert material" in most of our current mathematics courses. It's time to get rid of it and open the door to some fresh, important material.

My bet is that the underlying cause of this is our current fanaticism about having one-size-fits-all uniform texts chosen by central committees who often lack the expertise to make significant changes. They just go on tinkering with what was done the year before. It seems the central committees do not trust the initiatives of individual faculty members, so they shackle them with obsolete material. Publishers respond in kind. And the publishers stay away from texts for modern courses because new, modern, original texts are unlikely to sell well. This is the reason that most well-selling traditional calculus texts are clones of George Thomas's calculus course of the 1950s.

The trend is for engineering, biology and science departments to begin teaching the mathematics their students need. Mechanical engineering departments are teaching lots of advanced calculus and differential equations. Electrical engineering departments are teaching lots of probability and complex variables. According to a source inside the Stanford University Computer Science Department, they have decided:

- a. They'd like their students to have more math.
- b. But not the kind of math that's coming from the math department.
- c. It's never going to come from the math department.
- d. They'll start doing it themselves.

No wonder Sol Garfunkel and Gail S. Young wrote: "Our profession is in desperate trouble—immediate and present danger. The absolute numbers and the trends are clear. If something is not done soon, we will see mathematics department faculties decimated and an already dismal job market completely collapse. Simply put, we are losing our students." Are mathematics professors and departments in extreme denial? I wish SK had dealt with these serious issues.

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4. Specific Remarks about SK's Revision

SK: "We do not want our students to learn to push buttons. We want them to think critically and analytically. It has been argued that Mathematica and similar software to help students interact dynamically and visually with the graphics: Vary the value of a in the equation $y = ax^2 + bx + c$ and watch how the graph changes. That is not what I want my students to learn. I want them to understand that, for large values of x , the coefficient of a is the most important of the three coefficients. And changing its value affects the first and second derivatives in a certain way. And, in turn, these changes affect the qualitative behavior of the graph in a predictable fashion. AFTER [SK's emphasis] these precepts are mastered, the student may have some fun verifying them with computer graphics. BUT NOT BEFORE [my emphasis]."

Reaction: Why not before? Is this a moral issue? Certainly this is not an educational issue. Students (and research mathematicians) learn lots from examples. Here is my version: Have the students play with graphics, first varying a and coming up with a conjecture of what the influence of a is. Then ask them to explain why their response is correct. Then ask them to explain how changing the value of a affects the first and second derivatives in a certain way and how this is reflected in corresponding plots. In this way the students engage completely in Saunders MacLane's sequence for the understanding of mathematics: "intuition-trial-error-speculation-conjecture-proof." I don't care how many buttons students press; if it helps them to think critically and analytically, I'm all for it.

SK: In a programmed learning environment, whether the interface is a PC or with Mathematica Notebooks or with a MAC, the students cannot ask questions.

Reaction: Disagree. Students at computers can and do ask questions—lots of them.

SK: "What sense does it make to have a mathematics classroom, with a computer before each student, and the instructor delivering command to the students? ... People need to perform laboratory activities in their own time at their own pace."

Reaction: I agree thoroughly. A teacher-centered computer lab is absurd. The professor has to learn to relinquish total control when the students are in the lab.

SK: "Most of us were trained with the idea that the whole point of mathematics is to understand precisely why things work. To make the point more strongly, this attitude is what sets us apart from laboratory scientists."

Reaction: I agree and disagree. Throughout our courseware are two recurring themes:

1. One of the goals of mathematics is to explain why things work out the way they do.

2. There are no accidents in mathematics!

On the other hand, good mathematical research has always been a laboratory science. Top quality mathematical research invariably feeds off examples and special cases. With the computer, students can get lots of examples and begin to formulate what mathematical truth might be then go on to explain why—thereby engaging in the whole mathematical process. Students in lecture classes miss out on this opportunity. The lecturer handles all of this for them.

SK: “One of the highest and best uses of computer in mathematics instruction is as the basis for laboratory work.”

Reaction: I agree thoroughly. But I hasten to add that not all laboratory work is good. Weekly lab sections tacked onto an otherwise traditional course are of dubious value. The way the calculator is used in high schools to prepare for the AP exam is far from optimal.

SK: “Using the quadratic formula is easy. Analyzing a word problem is hard. A person who cannot do the first will probably not be able to do the second—with or without the aid of a machine.”

Reaction: Disagree. One does not follow from the other, as many students in Calculus&Mathematica “BioCalc” sections at Illinois have proved.

SK: “If you choose a poor text, you will have to pay for it through the semester.”

Reaction: Too many professors are not allowed to choose their texts. And how many really good texts are there out there?

SK: “...the respect a teacher must show his audience.”

Reaction: A really good point. But I am uncomfortable with the characterization of students as audience. Audiences are usually faceless and rarely participate; they just watch. The view of students as audience is one of the major defects of the lecture method.

5. Suggestions for Further Reading

Here are some sources for those who want to re-examine their ideas about teaching of mathematics:

a) Ralph Boas’s article “Can We Make Mathematics More Intelligible?” (*Amer. Math. Monthly* 88 (1981), 727–731) is a provocative short complement and counterpoint to SK’s revision. Samples are included in the text above. This and a number of Boas’s other essays were reprinted in the book “Lion Hunting & Other Mathematical Pursuits” (MAA,1995). Enlightening reading!

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b) Gian-Carlo Rota's book, *Indiscrete Thoughts* (Birkhäuser, 1997). Rota raises the right issues in the way only Rota can. A sample: "One must guard . . . against confusing the presentation of mathematics with the content of mathematics. An axiomatic presentation differs from the fact that is being presented as medicine differs from food. . . . Understanding mathematics means being able to forget the medicine and enjoy the food." If you read this book, you will not forget the experience!

c) Reuben Hersch's book, *What is Mathematics Really?*, (Oxford, 1997) A sample: "A philosophy of mathematics that obscures the teachability of mathematics is unacceptable."

d) H. Poincaré's books *Science and Hypothesis*, *The Value of Science*, and *Science and Method: Some Running Themes*: the value of intuition, arguing against reduction of mathematics to algebra à la Weierstrass; verification is not enough. Poincaré was one of the first to point out that mathematics teaching is not all it could be.

e) Henri Lebesgue's essays, *Measure and Integral* (Kenneth O. May editor, Holden-Day 1964). This book consists of pedagogical essays written by Henri Lebesgue and assembled by Kenneth O. May. The essays are very heavy on pedagogical and mathematical content with a passionate plea for better acceptance of decimal numbers in the mathematics classroom. Two samples: "There is a real hypocrisy, quite frequent in the teaching of mathematics. The teacher takes verbal precautions, which are valid in the sense he gives them, but that the students most assuredly will not understand the same way."

And:

"Unfortunately competitive examinations often encourage [an educational] deception. The teachers must train their students to answer little fragmentary questions quite well, and they give them model answers that are often veritable masterpieces and that leave no room for criticism. To achieve this, the teachers isolate each question from the whole of mathematics and create for this question alone a perfect language without bothering about its relationships to other questions. Mathematics is no longer a monument but a heap."