## 3.1 Personal Aspects

Like many activities in life, teaching is an intensely personal one. Some teachers have a lighthearted, informal, even jocular style. Others are more severe. Some give a rigid, structured lecture. Others conduct a Socratic interchange with the class. Some send students to the board to do problems (some, in the R. L. Moore style, do nothing but—see Section 1.12). Some instructors use overhead slides, computer simulations, symbolic manipulation software, and `Mathematica` graphics. Others do it all themselves, with just a piece of chalk. Some professors integrate a (computer-based) laboratory component into their courses. (In fact I would like to see mathematics become more of a "laboratory discipline". See Section 1.10.) All of these methods are correct. It is essential for you to be comfortable with your class. Therefore you should conduct the class in whatever fashion feels most natural to you.

However you should be willing to try new things. If you have never told a joke before, try telling a joke. If it works, you may be pleasantly surprised and may tell another. (But be forewarned: Eighteen year olds are insecure and are always worried that someone is making fun of them. Do not tell jokes that may be interpreted in that fashion. Do not tell jokes at anyone's expense. Do not tell sexist jokes. Do not use vulgar language or discuss offensive topics.) Try introducing the product rule with a story about how much trouble Leibniz had getting it right (see Section 1.7). Illustrate the subtleties of the constant of integration by integrating $\int 1/x\,dx$ by parts (without the constant) and deriving the assertion that $0 = 1$ :

$$\int \frac{1}{x}\,dx = \frac{1}{x}\cdot x - \int x\cdot\left(-\frac{1}{x^2}\right)\,dx = 1 + \int \frac{1}{x}\,dx$$

hence $0 = 1$. Some of these endeavors will fall flat. Others will breathe new life into an otherwise old and (for you) dull topic.

It is important to me that my classroom have the atmosphere of an interchange of ideas among intelligent people. I would be most uncomfortable to stand for an hour reciting a litany of abstract nonsense to a sea of blank faces. Thus I am continually trying new approaches, new angles, new ideas. It is a way to keep my classes fresh, even in a course that I have taught ten times before.

I usually do not find it useful to send students to the blackboard to do problems. First, the time that it takes for the student to get to the front of the room, falter around, and sit down again, is too great for the benefit obtained. I do everything myself because I can teach a great deal even while I am doing the most mundane example. But others have been sending students to the board for years and swear by it. Do what works for you.

I have no use for overhead projectors. To me, part of pacing a class is to let the material evolve on the blackboard. Part of the dynamic of my classroom style is moving back and forth in front of the material. But others find that they can be more organized if they write out the material in advance on overhead slides. Still others write the material in real time on the overhead slide. Yet another group writes very little, but stands in one spot and delivers a strictly oral lecture.

Remember that you are delivering a product. Cadillac does this differently from Mercedes Benz. You must develop your own delivery. The over-riding consideration is that you be comfortable with your classroom style so that you can in turn make your class comfortable. The style, organization, and content of your class is a reflection of you and your attitude toward the class. If you stand in front of your calculus class facing the blackboard, mumbling to yourself, and writing "Theorem–Proof–Theorem–Proof", then what message are you sending to the students? If instead you do a stand-up comedy routine and get around to the mathematics in the last ten minutes of class, then what message are you sending to the students?

Not everyone agrees with the spirit of what I am saying in this section. For example David S. Moore, in his essay written in acceptance of a major teaching award (see [MOO]), declaims

> Some teachers may have charisma. Not I. My image of a sound teacher is that of a skilled craftworker, a master machinist, say, who knows exactly what she must do, brings the tools she needs, does the work with straightforward competence, and takes pleasure in a job well done. She does her work right every day, and every day's work fits the larger plan of her project. The craftworker's skill is quite separate from her enthusiasm on that particular day, which, as C. S. Lewis said in another context, depends more on the state of our digestion than on any more cosmic influence.

This point of view is worth considering. To be sure, a journeyman machinist must reach and maintain a certain level of excellence. He cannot claim—two days per week—that he slept badly, or fought with his spouse, or was not in the mood. He is turning out precision milled parts for demanding applications, and his work must always meet a rather exacting standard.

We, as teachers, have a bit more slack. We are more like play-actors. Our days of divine inspiration can, and usually are, balanced out by days of doleful plodding. I think that Moore's remarks can inspire us to set a basic standard to which we all can, and should, aspire. There really should be no lecture or class that we give that is less than competent, less than accessible, less than accurate. That is not too much to ask.

But the machinist is working with an inert substance—such as stainless steel. And his task is essentially a zero-one game. He either produces his precision part correctly or he doesn't. Students are *not* inert substances. Far from it, they are highly volatile. And our task—to impart knowledge, to excite, to set an example of learning and scholarship—is hardly a zero-one game. It is a multi-parameter process that requires skill and care to do well.

Teaching is an intensely human activity. Part of our job is to convey facts. But perhaps a more important part is to inspire and to ignite curiosity. If we want to influence students to be scientists, to be math majors, to be thinkers, to get caught up in mathematics, then we must be more than competent. We must be on fire ourselves. So let Moore's criteria be a lower bound. Let the heavens be an upper bound.

And now a coda on haberdashery. Speaking strictly logically, it should not make any difference whether you wear a suit, or jeans and a work shirt, or wear a loin cloth and carry a spear when you teach. But it does. Dressing nicely sends a subtle signal to the class that you are the person in charge. If you wear ripped, stone-washed jeans, a tie-died tee shirt, and have long, greasy hair tied in a bandana, then you may convey to the class that you just came from lube-ing your car, or that you have great empathy with sharecroppers, or that you are just plain "folks". But you may, inadvertently, also convey that you just don't give a damn.

Straightening your tie and combing your hair before going to class is like putting on your mortar board before going to graduation. You are pausing to say to yourself, "Now I am going off to do something important."

## 3.2 Attitude

I have long felt that those who cannot teach are those who do not care about teaching. If you actually care about transmitting knowledge and inspiring curiosity and a love for learning, then much of what I say in this book follows automatically. But some comments should be made.

Your students are a lot like you. When you enroll for a class, you have certain expectations. It is reasonable, therefore, that when you *teach* a class you should endeavor to live up to those same expectations. From this it follows that you should prepare, be organized, be fair, be receptive to questions, meet your office hour, and so forth.

On the other hand, your students are not like you. Especially in elementary courses, you cannot expect your students to be little mathematicians. Many of them are in the class *only* because it is a prerequisite for their majors. Try to remember how you felt when you took anthropology or Latin or biology. Not everyone has a gift for mathematics. Unfortunately, some people have an attitude problem to boot (this attitude problem is sometimes termed "math anxiety"—see Section 3.6).

So you must learn to be sympathetic and receptive, and you must learn to be patient. Teaching is part of your craft, and part of your job. Perhaps if you are Gauss, or if you have just proved the Riemann hypothesis, then you can justifiably say that you are above these pedestrian considerations about teaching. I'm betting that you are not either of these. If you call yourself a professor, and if you have the temerity to stand in front of an audience and profess, then you should show your audience some respect and consideration.

David S. Moore, an award-winning teacher already quoted in Section 3.1, noted the following when addressing an audience of teachers (see [MOO]) on the craft of good teaching:

> Our individual experience, both as students and later as teachers, is atypical. As students, we were the survivors, the fittest by some quite esoteric standards of fitness.

It would be heartless to say to a manic depressive, "Just cheer up," or to say to a drug addict, "Just say no." Likewise, it is heartless to tell a person who thinks he has math anxiety that in fact he is wrong—he is just a lazy bum. At the same time, mathematics instructors are not trained to treat math anxiety, any more than they are trained to treat nervous disorders or paranoia. If a student told you that he had dyslexia, then you certainly would not try to treat it yourself; nor would you tell the student that he just didn't have the right attitude, and should work harder. Likewise, if one of your students complains of math anxiety, you should take the matter seriously and realize that you are not qualified to handle it. Refer that student to a professional. Most every campus has one.

Never forget that you are a powerful figure in your students' lives. This fact carries with it a great deal of responsibility. If you were a follower of Dr. Kervorkian,[3] then you might take a troubled student in hand and say, "I know you are doing poorly in your math class. You must be in a great deal of pain, and suffering from shame. I have a solution for you—it's rather permanent, but it's painless." Chuckle if you will, but this is no joke. Problems of the psyche can be severe and dangerous. If a student comes to you with psychological problems then make sure that he gets help from someone who is qualified to administer that help.

Unfortunately, at some schools the "math anxiety" thing has gotten way out of hand. There are good universities where a student may be excused from a mathematics or statistics course (one that is *required* for his major) by simply declaring himself to be math phobic, or possessed of math anxiety. It is a sad state of affairs, but there is nothing that we math teachers can do about it. Because you and I are, by nature, good at mathematics, and because we do not suffer from math anxiety, it is difficult for us to empathize with people who suffer from this malady. It is really best to let those who know the literature, know the symptoms, and know the treatments to handle students who have this form of stress. Do not hesitate to refer your students to the appropriate counselor when the situation so dictates.

## 3.7 How Do Students Learn?

Psychologists, sociologists, education theorists, and anthropologists have debated the nature of the learning mechanism for decades. There is no general agreement on how students learn. See [ASI] for a modern discussion of some of the issues. This is a question on which serious scholars will spend their entire careers—and still not reach definitive conclusions. My only intent here is to share with you a few personal perceptions.

It is my opinion that the very best students tend to teach themselves. The instructor points out signposts for such a student, and then the student's native intellect takes over.

[3]Dr. Jack Kervorkian is the physician who has garnered considerable notoriety for assisting the suicides of severely ill people.

On the other hand, weak students are often quite dependent on the instructor and (perhaps) the text and the lectures. If you agree that these people are worth teaching at all, then you must be there for them. Provide good lectures and a reasonable text for them to work with. Set a good pace. Answer their questions. Be as helpful, and as encouraging, as possible to students who have the courage to come to your office for help.

Students of middling abilities are perhaps in the majority, and they share properties with both the best students and the worst. Know with certainty that you cannot please everyone. Students in the middle don't want to be clobbered with abstract theory, and they don't want to be bored with trivialities. It is not very difficult to follow your intuition and find a pitch that is appropriate for the middle level of a class. What is trickier is to do something for the lower end and for the upper end—and to handle the vast spectrum of student abilities all at the same time!

I like to pepper my classes with what I call "Culture Spots". I present one of these perhaps once or twice per week. These are tempting little bits that I throw out for the brightest, and the more curious, students to think about. If a culture spot sparks anyone's interest, then he can see me after class or in my office hour and we can discuss it further. An example of a culture spot is this: When I teach multivariable calculus, I of course must give the important example of the vector field

$$\mathbf{v}(x,y) = \left( \frac{-y}{x^2+y^2} , \frac{x}{x^2+y^2} \right) ,$$

which is defined on the annulus $\{(x,y) \in \mathbb{R}^2 : 1 < x^2 + y^2 < 4\}$, is closed, but is not conservative. Usually there is some student who asks me whether this is the only example. Great question! So I write "Culture Spot" on the blackboard, point out that this is an advanced topic that I'm only going to mention, and then tell them that one of the great theorems of twentieth-century mathematics is de Rham's theorem. I go on to say that de Rham's theorem says that the number of examples is equal (roughly speaking) to the number of holes in the domain of definition. Later on, when questions about different types of holes come up, I can refer to de Rham's theorem, and also make allusions to homotopy and homology theory.

Note that these advanced ideas are presented very briefly, and in such a fashion that students know that these are ancillary remarks, not part of the course, and are nothing that they will ever be examined on. But these are an entree for the gifted students to come talk to me, and to give them something to think about.

If you ever wonder why a calculus teacher should have a Ph.D. and be a research mathematician, then think about what I have said in the last two paragraphs. An instructor with minimal training from a junior college might be qualified to teach integration by parts to non-majors. But could he ever provide the sorts of insights that I have just described? I'll leave it to you to provide the answer.

Thus the lesson is to strive at least to provide some stimulation for students at all levels. The ability to do so, without an unreasonable amount of effort on your part, can come only with experience and determination. Even if you had

a class with just three students, it is likely that their levels of ability would fall into two or more disparate categories. Thus there is no escaping the realities of having an uneven audience. Being forewarned, and being thoughtful, can help you to present your course so that you teach something of value to most of the students most of the time.

Let me cast these matters in a different light. An instructor who is demanding and difficult and who omits many details from his lectures will challenge the talented students and force them to go off and learn the material on their own. Certainly that was the nature of the graduate program that I attended, and it did me a world of good. On the other hand, an instructor who is lucid and who proceeds at a comfortable pace makes everything look too easy and can lull students into a false sense of security. That instructor will also bore the gifted students. How do you address both of these phenomena in your classroom?

Because at most universities we are training a widely diverse group of students, the issues raised in the last paragraphs are unavoidable. One solution, in a fundamental course like calculus for example, is for the math department to set up several tracks. There is a calculus course for scientists and engineers, one for pre-medical students, and one for business students (some colleges even have a "calculus for poets" course). This is a commonly used method to slice up student abilities so that the spectrum in any given classroom is not so broad. Another approach to the issue of widely varying student abilities might be to rethink the traditional classroom/lecture format and divide students into working groups in which they can seek their own level. The final word has not been said on how to deal with these problems. See Section 2.5 for more on the value of group work.

We should continue to explore new methods to teach mathematics. We are constantly hounded by certain observed facts. The failure rate in the large lecture system is unacceptably high. The retention rate is unacceptably low. America appears to be falling behind in the technology race because we are not training our young people thoroughly and well. Can we find a more effective way to teach—one that addresses the perceived problems?

In my view, it is simplistic to lay the blame for our teaching problems on the *formal method* by which we have been teaching mathematics. No methodology is perfect. But we all know something about how to lecture. We should consider how to make it a more useful tool before we discard it altogether and adopt new, unfamiliar tools.

Let us own up to the fact that many of us—especially those trained at high-powered research departments—are often not trained to care about teaching. Many of us do not. That is not the way the value-reward system is set up. Certainly none of us has had any real training in how to lecture, or how to get students engaged in the learning process, or how to teach discourse. The traditional methods of teaching still have much to offer, provided that they are being used by people who are properly trained and who care. But learning to teach is a lifelong process, and we should be open to learning new methods of teaching, and also to new facets of the old, familiar methods of teaching.

An instructor who stands in front of his class and drones on in an uninflected monotone, copying dry, discursive mathematical facts and theorems and proofs onto the blackboard is not teaching. He is not doing his job. In fact he is simply

*hiding behind the mathematics.* He does not have the courage to face his students and to interact with his students.

I once had a colleague who so detested his students and his teaching assignment that he would handle his class as follows. Three minutes before the class period, he would rip the relevant pages from his copy of the textbook. At the appointed moment he would storm into the classroom, copy the material from the eviscerated pages directly onto the blackboard, ignore all student questions, and storm out of the room when he was finished. (I am happy to say that when the chairman got wind of this behavior he yanked this tenured, full professor out of his class—permanently—and docked his salary.) You may find this behavior so bizarre as to be ludicrous. But if you examine your conscience carefully then you will have to admit that we are all guilty of this sort of behavior from time to time. On days when things are not going well, or you've been selected for an IRS audit, or you cannot get interested in the topic of the day, or you cannot figure out any good way to explain it, you just show up for class and blow the whole thing off. You hide behind the mathematics, *and you do not teach.* Your class is less like a learning experience and more like Kabuki theater.

So my message is a simple one. Stop hiding behind the mathematics. Don't treat the mathematics as a screen or a foil. Step around and get out in front of the mathematics, where the students are. Stand beside them and look at the mathematics with them. Point out the interesting features. Help them to appreciate what they see. Explain it. Talk to them about it.

Your class will be more involved with, and more engaged in, the learning process if each class meeting contains activities in which the students can *participate.* If you copy a dry lecture onto the blackboard, discourage questions, and then just walk out of the room at the end, then you have not offered the students such activities. If, instead, you engage the students in discourse (Section 3.14), then you have begun to break the ice. If you break the students up into discussion groups, then you have gone even further to enable student activities. If you send the students to the blackboard, either to talk to the class or to work problems, then you have gone further still. The ideas in this paragraph are another illustration of the precept that in order to be a successful instructor you must be consciously aware of what it is you are trying to accomplish.

Some universities these days are telling their faculty, "Don't be the 'sage on the stage'. Instead be the 'guide on the side'." I am sorry to say that for most university administrators these words do not spring from some deep well of philosophical angst. Rather, they are seeking ways to teach more students with fewer faculty. Even so, there may be something of value in this adage. You, the instructor, should not be like the Wizard of Oz (see [BAU]), hiding behind a screen and barking out profundities. Instead, you should be more like Dorothy, standing with the students, helping them to ask questions and seek answers. (See also George Andrews's Appendix on being a sage on the stage.)