Answer the questions in the spaces provided on the question sheets. Please be sure to show ALL work and justifications so you can receive partial credit. Good luck!

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Name: KEY
1. The marginal cost of producing $x$ calculus textbooks is $2.5x + 75$, and the initial cost of production is 2,000.

(a) (5 points) What is the cost of producing $x$ units?

\[
C(x) = \int (2.5x + 75) \, dx
\]

\[
= \frac{2.5}{2} x^2 + 75x + K
\]

\[
C(0) = 2000 = \frac{2.5}{2} (0)^2 + 75 \cdot 0 + K = K
\]

\[
C(x) = \frac{2.5}{2} x^2 + 75x + 2000
\]

(b) (5 points) Find the average cost of producing from 0 to 100 units. You do not need to evaluate your answer.

\[
\text{Average} = \frac{1}{100-0} \int_0^{100} \left( \frac{2.5}{2} x^2 + 75x + 2000 \right) \, dx
\]

\[
= \frac{1}{100} \left( \frac{2.5}{6} x^3 + \frac{75}{2} x^2 + 2000x \right)^{100}_0
\]
2. When I play golf, I tend to hit at least one ball into the water. Fortunately, I use biodegradable golf balls that decay exponentially.

(a) (5 points) If it takes the ball 1 year to decay by 75%, what is the rate of decay of the golf ball?

\[
0.25P = P e^{-\lambda}
\]

\[
0.25 = e^{-\lambda}
\]

\[
\lambda = \ln(0.25)
\]

(b) (5 points) At what time \( t \) will the ball have only 10% left?

\[
0.10P = P e^{-\lambda t}
\]

\[
0.1 = e^{-\lambda t}
\]

\[
\ln(0.1) = \ln(0.25) t
\]

\[
t = \frac{\ln(0.25)}{\ln(0.1)}
\]
3. Evaluate the following definite integrals using any technique you want:

(a) (10 points) \[ \int_{-1}^{1} - \sqrt{9 - 9x^2} \, dx = -3 \int_{-1}^{1} \sqrt{1 - x^2} \, dx = \frac{\pi}{2} \]

(b) (10 points) \[ \int_{1}^{e} x^2 e^{5x} \, dx = 0 \] since bounds are the same.

(c) (10 points) \[ \int_{1}^{3} \frac{\ln(2x)}{x} \, dx \]

\[ u = \ln(2x) \]
\[ du = \frac{2}{2x} \, dx \]

\[ \int \frac{u}{\ln 2} \, du = \frac{1}{2} u^2 \bigg|_{\ln 2}^{\ln 6} \]
\[ = \frac{1}{2} \left( \left( \ln 6 \right)^2 - \left( \ln 2 \right)^2 \right) \]

\[ = \frac{1}{2} \left( \ln(2x)^2 \right) \bigg|_{1}^{3} \]
4. Let \( f(x) = x - 1 \) and \( g(x) = 1 \).

(a) (5 points) Graph \( f(x) \) and \( g(x) \).

(b) (5 points) Find the area between \( f(x) \) and \( g(x) \) on the interval for \(-3 \leq x \leq 3\) using geometric formulas.

\[
\text{Area} = \frac{25}{2} + \frac{1}{2} = 13
\]

(c) (5 points) Find the area between \( f(x) \) and \( g(x) \) on the interval for \(-3 \leq x \leq 3\) by setting up an integral. Do NOT go past setting up the integral.

\[
\int_{-3}^{2} (1 - (x-1)) \, dx + \int_{2}^{3} ((x-1)-1) \, dx
\]

\(\int \) \quad + \quad \text{splitting}

\(\int\) order I

\(\int\) order II
5. (10 points) Find \( \int \frac{3x + 2}{x^2 + x} \, dx \)

\[
\frac{3x + 2}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}
\]

\[
3x + 2 = A(x + 1) + Bx
\]

\(x = 0 \Rightarrow 2 = A\)

\(x = -1 \Rightarrow -1 = -B \Rightarrow B = 1\)

\[
\int \left( \frac{2}{x} + \frac{1}{x+1} \right) \, dx
\]

\[
= 2\ln|x| + \ln|x+1| + C
\]
6. Find \( \int x \sqrt{x+1} \, dx \) in the following two ways:

(a) (5 points) Using integration by parts.

\[
\begin{align*}
&u = x \\
du = dx \\
&v = \frac{2}{3} (x+1)^{\frac{3}{2}} \\
dv = \sqrt{x+1} \, dx
\end{align*}
\]

\[
\begin{align*}
x \cdot \frac{2}{3} (x+1)^{\frac{3}{2}} - \int \frac{2}{3} (x+1)^{\frac{3}{2}} \, dx \\
= x \cdot \frac{2}{3} (x+1)^{\frac{3}{2}} - \frac{4}{15} (x+1)^{\frac{5}{2}} + C
\end{align*}
\]

(b) (5 points) Using a \( u \), \( du \) substitution.

\[
\begin{align*}
u &= x+1 \\
du &= dx
\end{align*}
\]

\[
\begin{align*}
&= \int (u-1) \sqrt{u} \, du \\
&= \int (u^{\frac{5}{2}} - u^{\frac{3}{2}}) \, du \\
&= \frac{2}{5} u^{\frac{5}{2}} - \frac{2}{3} u^{\frac{3}{2}} + C \\
&= \frac{2}{5} (x+1)^{\frac{5}{2}} - \frac{2}{3} (x+1)^{\frac{3}{2}} + C
\end{align*}
\]
7. Find the following integrals:

(a) (5 points) \( \int xe^x \, dx \)

\[ u = x \quad v = e^x \]
\[ du = dx \quad dv = e^x \, dx \]

\[ = xe^x - \int e^x \, du \]
\[ = xe^x - e^x + C \]

(b) (10 points) \( \int x^3 e^{x^2} \, dx \) [Hint: Use part (a).]

\[ u = x^2 \]
\[ du = 2x \, dx \]

\[ = \frac{1}{2} \int ue^u \, du \]
\[ = \frac{1}{2} \int ue^u \, du \]
\[ = \frac{1}{2} (ue^u - e^u) + C \]
\[ = \frac{1}{2} (x^2 e^{x^2} - e^{x^2}) + C \]