Last time we started discussing infinite limits. Let's start by analyzing some vertical asymptotes. Remember

\[ x = a \] is a vertical asymptote of \( f(x) \)

if \[ \lim_{x \to a^-} f(x) = \pm \infty \] and \( \lim_{x \to a^+} f(x) = \pm \infty \)

If \( f(x) = \frac{p(x)}{q(x)} \) is a rational function, we can tell where the vertical asymptotes are by simplifying and seeing what values of \( x \) make the denominator 0. But how do we calculate the left and right limits as \( x \) approaches these asymptotes?

Ex: \[ \lim_{x \to 3} f(x) \] where \( f(x) = \frac{x - 2}{x - 3} \).

Based on experience, we know there is a VA at \( x = 3 \).
If \( \lim_{x \to 3^-} f(x) = \infty \), then \( f(x) \) should definitely be positive close to 3 on the left.

If \( \lim_{x \to 3^+} f(x) = -\infty \), then \( f(x) \) should definitely be negative close to 3 on the left.

So let's try plugging values in close to 3 on the left.

Same for \( \lim_{x \to 3^+} f(x) \). So the idea is to test points near the limit and see if it is positive or negative.
So, now we know what happens at asymptotes and can find holes. But what happens at the ends of our graphs? Now we want to be able to understand the "end behavior" of a function $g(x)$.

**Ex** What is the end behavior of $\frac{1}{x}$?

![Graph](image)

As we go to the ends of our graph, $\frac{1}{x}$ goes to 0.

We say $\lim_{x \to \infty} f(x) = L$ if $f(x)$ gets as close to $L$ as we want if we go far enough to the right. Same for $\lim_{x \to -\infty} f(x) = L$ but to the left.

If $\lim_{x \to \infty} f(x) = L$ for some number $L$, we say $f(x)$ has a horizontal asymptote at $y = L$. 
Ex \( \lim_{x \to -\infty} \left( 5 - \frac{2}{x^3} \right) \)

Ex \( \lim_{x \to \infty} \left( 2 + \frac{\sin x}{\sqrt{x}} \right) \)

However, not every function has horizontal asymptotes. Examples? How about \( h(x) = x^2 \). But if \( \lim_{x \to \infty} h(x) \) does not exist, what could it be? \( \pm \infty \)! Just like vertical asymptotes but in reverse.

Ex \( j(x) = x^n \), \( k(x) = \frac{1}{x^n} \), \( n > 0 \)

Now, what about polynomials?

Ex \( p(x) = 2x^3 - 3x^2 + 6 \)
Rational Functions?

\[ \frac{x+2}{x^2-1} \quad \frac{10x^4+100x^3-2}{5x^4-\pi x+3} \]

\[ \frac{x^3-10^8x+10^x}{2x+\sin(1)} \]

In the end, it's the leading terms (the largest powers of \(x\)) that determine the end behavior of the rational function. It's the same idea for similar functions as well that introduce radicals.

\[ \frac{10x^2+2}{\sqrt{25x^4+1}} \]

And in general, there are a couple other limits we should know for \(e^x\), \(\ln x\), \(\cos/sin\).