Recall, if we have a right triangle

\[ \text{hypothenuse} \rightarrow h \]
\[ \theta \]
\[ \frac{\pi}{2} - \theta \]
\[ a = \text{adjacent} \]
\[ o = \text{opposite} \]

we can define a number of trigonometric functions:

\[ \sin \theta = \frac{o}{h} \]
\[ \cos \theta = \frac{a}{h} \]
\[ \tan \theta = \frac{o}{a} \]

(SOHI CAH TOA)

Note: \[ \tan \theta = \frac{o}{a} = \frac{o/h}{a/h} = \frac{\sin \theta}{\cos \theta} \]

Also, note that
\[ \sin \left( \frac{\pi}{2} - \theta \right) = \cos \theta \]
\[ \cos \left( \frac{\pi}{2} - \theta \right) = \sin \theta \]

which explains the graphs just being shifts of each other.
Inverse trig functions

\[ y = \sin^{-1} x \quad \text{if} \quad x = \sin y \]

\[ y = \cos^{-1} x \quad \text{if} \quad x = \cos y \]

\[ y = \tan^{-1} x \quad \text{if} \quad x = \tan y \]

\[ \text{Ex} \quad y = \sin^{-1} \frac{\sqrt{3}}{2} \]
\[ y = 60^\circ = \frac{\pi}{3} \]

\[ \text{Ex} \quad y = \cos^{-1} \frac{\sqrt{2}}{2} \]
\[ y = 45^\circ = \frac{\pi}{4} \]

\[ y = \sin^{-1} \left( -\frac{\sqrt{3}}{2} \right) \]
\[ y = -\frac{\pi}{3} \]

\[ y = \cos^{-1} \left( -\frac{\sqrt{2}}{2} \right) \]
\[ y = -\frac{\pi}{4} \]
Ex \[ \cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = \frac{\sqrt{3}}{2} \]

\[
\begin{array}{cccc}
\sin^2\theta & 1 & \sqrt{3} \\
2 & \text{ } & \text{ }
\end{array}
\]

**Other trig functions**

\[ \csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x} \]

**Important things to remember**

\[ \cos^2 \theta + \sin^2 \theta = 1 \quad \text{Pythagorean theorem} \]

So the points \((\cos \theta, \sin \theta)\) lie on a circle.
Derivatives of Trig Functions

First, we will need a certain limit:

\[
\lim_{x \to 0} \frac{\sin x}{x}
\]

Consider the following picture:

Looking at areas:

\[
\frac{\pi}{2} \leq \frac{x}{2} \leq \frac{\pi}{2} \\
\frac{1}{2} \cos x \sin x \leq \frac{x}{2} \leq \frac{1}{2} \frac{\sin x}{\cos x}
\]

\[
\Rightarrow \cos x \leq \frac{x}{\sin x} \leq \frac{1}{\cos x}
\]

Using the squeeze theorem implies

\[
\lim_{x \to 0} \frac{\sin x}{x} = 1
\]
\[ \lim_{x \to 2} \frac{\sin(x-2)}{x^2-4} \]

\[ \lim_{x \to 0} \frac{\sin 3x}{x} \quad \lim_{x \to 0} \frac{\sin 6x}{\sin 2x} \]

This limit is a necessary part in the proof of (which you should read)

\[ \frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \]

Of course, now we can use the product and quotient rules for all kinds of functions involving \( \sin x \) and \( \cos x \).

\[ \frac{d}{dx} (\tan x) = \sec^2 x \]

\[ \frac{d}{dx} (\csc x) = -\csc x \cot x \]

\[ \frac{d}{dx} \left( \frac{\tan x}{1 + \sec x} \right) \]
Higher derivatives

If \( f(x) \) is a differentiable function, we say \( f'(x) \) is the first derivative of \( f \). If \( f'(x) \) is differentiable, we say \( f''(x) = (f')'(x) = f''(x) \) is the second derivative of \( f \).

\[ f'''(x) = 3 \text{rd derivative}, \text{ etc.} \]

Ex. If \( f(x) = \sin x \)

\[ f'(x) = \frac{d}{dx} \sin x = \cos x \]

\[ f''(x) = \frac{d}{dx} \cos x = -\sin x \]

\[ f'''(x) = \frac{d}{dx} (-\sin x) = -\cos x \]

\[ f^{(4)}(x) = \frac{d}{dx} (-\cos x) = \sin x \]

So \( f(x) = f^{(10)}(x) \)

Ex. What is \( f^{(10)}(x) \)?