MA 123
Section 3.5
2/25/16

Just for more practice:

1) \( \frac{d}{dx} (\sec x \ csc x) \)

2) \( \frac{d}{dx} \left( \frac{\cot x}{1 + \csc x} \right) \)

3) \( \lim_{x \to \frac{\pi}{2}} \frac{\cos x}{x - \frac{\pi}{2}} - \lim_{x \to \frac{\pi}{2}} \frac{\sin(\frac{\pi}{2} - x)}{-\left(\frac{\pi}{2} - x\right)} \)

Section 3.7

So far, we have looked at how a variable function \( y \) changes with respect to \( x \).

But what if there are more pieces involved?

Ex: Suppose Yvonne, Uri, and Xanadu all take a test. Yvonne is 2 times as fast as Uri, and Uri is 3 times faster than Xanadu.
How fast is Yvonne relative to Kwanalu?

\[ \frac{dy}{du} = 2, \quad \frac{du}{dx} = 3, \quad \text{so} \quad \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 6 \]

relative change

This process of finding \( \frac{dy}{dx} \) with an intermediate variable is using what's called the chain rule.

Alternatively, we can view it as outer function:

\[ y = f(u) \quad \text{and} \quad u = g(x), \quad \text{so} \quad y = f(g(x)) \]

so \( \frac{dy}{du} = f'(u) \quad \frac{du}{dx} = g'(x) \) and

\[ \frac{dy}{dx} = f'(u) g'(x) = f'(g(x)) g'(x) \]
Ex 1) \( \sin^3 x \)  
Let \( f(u) = u^3 \), \( g(x) = \sin x \)  
\[ = (\sin x)^3 \uparrow \quad \text{outside} \]  
\[ \downarrow \quad \text{inside} \]

2) \( \sin \sqrt[3]{x} \)  
Let \( f(u) = \sin u \), \( g(x) = x^3 \)  
\[ \text{outside} \quad \text{inside} \]

3) \( \frac{1}{3} \frac{d}{dx} (6x^3 + 3x + 1)^{10} = 10 (6x^3 + 3x + 1)^9 \cdot (18x^2 + 3) \)

4) \( \frac{d}{dx} \sqrt[3]{\tan x + 10} = \frac{1}{2} (\tan x + 10)^{-\frac{1}{3}} \cdot \sec^2 x \)

Remember, these rules aren't in isolation. Sometimes you have to combine.

Ex 1) \( \frac{d}{dx} (e^{\cos^3 x}) = e^{\cos^3 x} \cdot \frac{d}{dx} (\cos^3 x) \) 

repeated use of chain rule  
\[ = e^{\cos^3 x} \cdot 3 \cos^2 x \cdot \frac{d}{dx} (\cos x) \] 

\[ = e^{\cos^3 x} \cdot 3 \cos^2 x \cdot (-\sin x) \]
2) \( \frac{d}{dx} \sin (\sin (e^x)) \)

3) \( \frac{d}{dx} \left( x^2 \sqrt{x^2 + 1} \right) \)

4) \( \frac{d}{dx} \left( \frac{\sin^4 x}{1 + \cos^2 x} \right) \)