Last time we introduced the definite integral $\int_a^b f(x) \, dx$ that measures the signed-area between $f(x)$ and the $x$-axis on the interval $[a, b]$.

![Graph showing positive and negative areas]

**Example:**

- $\int_a^b f(x) \, dx$  
- $\int_b^c f(x) \, dx$  
- $\int_c^a f(x) \, dx$

Areas:
- $A = 10$
- $B = -6$
- $C = 4$
Last important properties of $\int_a^b f(x) \, dx$

1) $\int_a^b k \cdot f(x) \, dx = k \int_a^b f(x) \, dx$

2) $\int_a^b (f(x) + g(x)) \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx$

Just like indefinite integrals!

Section 5.5

Oh, so now how do we find the area if we can't resort to geometric formulas for rectangles and triangles?

Ex

\[
\int_0^2 f(x) = 1 \cdot (2-0) = F(2) - F(0)
\]
Wait, \[ f(x) = F'(x) = \frac{F(2) - F(0)}{2 - 0} \]

and \[ \int_0^2 f(x) \, dx = f(x) \cdot (2-0) \]

\[ = \frac{F(2) - F(0)}{2 - 0} \cdot (2-0) \]

\[ = F(2) - F(0) \]

In this case, \[ \int_a^b f(x) \, dx = \text{change in the antiderivative of } f(x). \]

On top of that, it didn’t matter which antiderivative we used. We’ll come back to this in a moment.

Ex. \( g(x) = -x^2 + 3x \), \( 0 \leq x \leq 1 \)

\[ \begin{align*}
\int_0^1 g'(x) \, dx &= \text{Check:} \\
\int_0^1 g'(x) \, dx &= g(1) - g(0).
\end{align*} \]
It works! In general, it does as long as we can find an anti-derivative!

This is known as the fundamental theorem of calculus:

If $f$ is continuous on $[a, b]$, and $F$ is any antiderivative of $f$, then

$$\int_a^b f(x) \, dx = F(b) - F(a)$$

And we don't need the $+C$!

$$F(b) + C - (F(a) + C) = F(b) - F(a)$$

Ex: $\int_{\phi}^{3} \left( x^3 + 4e^x - \frac{10}{x} \right) \, dx$