Last time we introduced the Fundamental Theorem of calculus:

$$\int_{a}^{b} f(x) \, dx = F(b) - F(a),$$

where $F(x)$ is an antiderivative of $f(x)$.

Example:

$$\int_{0}^{2} \frac{x}{x^2 + 1} \, dx$$

Let $u = x^2 + 1$, then $du = 2x \, dx$

$$= \int_{0}^{2} \frac{1}{2} \frac{du}{u}$$

$$= \frac{1}{2} \ln |u| \bigg|_{0}^{2}$$

$$= \frac{1}{2} \ln (2^2 + 1) - \frac{1}{2} \ln 1$$

$$= \frac{1}{2} \ln 5 - \frac{1}{2} \ln 1$$

$$= \frac{1}{2} \ln 5$$
Now for a more practical example:

Ex Let \( W(x) \) = the weight of a newborn. Doctors have determined the child grows at a rate of \( W'(x) = 0.25x + 0.92 \). If \( x \) is measured in months and the baby is initially 8 pounds, how much does the baby weigh after 6 months?

\[
W(6) = W(0) + \int_0^6 W'(x) \, dx
\]

\[
= 8 + \int_0^6 (0.25x + 0.92) \, dx
\]

\[
= 8 + \left[ 0.125x^2 + 0.92x \right]_0^6
\]

\[
= 8 + 10
\]

\[
= 18
\]
If \( s(t) \) is position, then \( V(t) = s'(t) \) is velocity. So, change in position between times \( a \) and \( b \) is

\[
s(b) - s(a) = \int_a^b V(t) \, dt
\]

But the average velocity was

\[
V_{av} = \frac{s(b) - s(a)}{b - a}.
\]

So

\[
V_{av} = \frac{1}{b - a} \int_a^b V(t) \, dt.
\]

In general, the average value of a function \( f \) on \([a, b] \) is

\[
\frac{1}{b - a} \int_a^b f(x) \, dx.
\]

Ex If \( A(t) = 1000 e^{0.02t} \) is the amount of savings I have after \( t \) years, then the average amount I have over 10 years is

\[
\frac{1}{10} \int_0^{10} 1000 e^{0.02t} \, dt = 100 \left( \frac{e^{0.2}}{0.02} \right) \Big|_0^{10}
\]

\[
= 5000 \left( e^{0.2} - 1 \right)
\]

\[
= \$1107
\]
Area between curves = difference of integrals

\[ \int_{a}^{b} (\text{top} - \text{bottom}) \, dx \]

Ex

Area between \(5 - x^2\) and \(2 - 2x\)

Ex

\( x^2 - x \) and \( 2x \) on \([-2,3]\).