Quiz 5  
MA 242, LINEAR ALGEBRA, SPRING 2016  
SECTION: C1  
NAME: KEY

Problem 1. Let \( A = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -1 & 4 & -2 \end{bmatrix} \). (1 pt) for signs

a) (4 points) Find the determinant of \( A \) using the cofactor method.

\[
\det A = 0 \begin{vmatrix} 2 & 1 \\ 4 & -2 \end{vmatrix} + 3 \begin{vmatrix} 7 & 1 \\ -1 & -2 \end{vmatrix} - (-1) \begin{vmatrix} 7 & 2 \\ -1 & 4 \end{vmatrix}
\]

\[
= 0 + 3(7(-2) - (-1)(1)) + 1(7(4) - (-1)(2))
\]

\[
= 3(-14) + 1(-30)
\]

\[
= -9 \quad \text{(2 pts) for correct setup}
\]

(1 pt) for answer

b) (4 points) Find the determinant of \( A \) using the reduction method.

\[
\begin{vmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -1 & 4 & -2 \end{vmatrix} = -\begin{vmatrix} -1 & 4 & -2 \\ 0 & 3 & -1 \\ 7 & 2 & 1 \end{vmatrix} = -\begin{vmatrix} -1 & 4 & -2 \\ 0 & 3 & -1 \\ 0 & 0 & -3 \end{vmatrix}
\]

\[
= -\begin{vmatrix} -1 & 4 & -2 \\ 0 & 3 & -1 \\ 0 & 0 & -3 \end{vmatrix} = (-1)(3)(-3) = -9
\]

(1 pt) for sign change  
(2 pts) for reduction  
(1 pt) for same answer as a)

c) (2 points) Is \( A \) invertible?

Yes, since \( \det A \neq 0 \)
Problem 2. Suppose $A$ and $B$ are $3 \times 3$ matrices with $\det A = 2$ and $\det B = 5$. Find the following values:

a) (2 points) $\det B^4 = (\det B)^4 = 5^4 = 625$

b) (2 points) $\det A^T = \det A = 2$

c) (3 points) $\det A^{-1}B = \det A^{-1} \cdot \det B \in (1 \text{ pt})$

\[ = \frac{1}{\det A} \cdot \det B \]

\[ = \frac{5}{2} \in (1 \text{ pt}) \]

d) (3 points) $\det 3AB^T$

\[ = 3^3 \det AB^T \]

\[ = 27 \det A \det B \] (1 pt) for \[ \det B^T \]

\[ = 27 \cdot 2 \cdot 5 \]

\[ = 270 \] (1 pt)