Complementarity of one-particle and two-particle interference

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In a two-particle interferometer, one can study the variation of both single- and joint-detection probabilities as functions of the phase shifts of the beams. By combining the usual definition for one-particle fringe visibility v_i $(i=1,2)$ with a reasonable proposed definition for two-particle fringe visibility v_{12}, we show that $v_i^2+v_{12}^2 \leq 1$ or, equivalently, $v_i v_{12} \leq \frac{1}{2}$. Some extensions are discussed.

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I. INTRODUCTION

In the last decade the techniques of standard one-particle interferometry have been extended to beams containing pairs of particles in quantum-mechanically-entangled states, i.e., states that cannot be expressed as products of two one-particle states. There have been numerous theoretical analyses and experiments (e.g., [1–5])—the latter all performed with photon pairs. A schematic arrangement is given in Fig. 1. From the source *S*, a pair of particles $1+2$ emerges, one of which propagates in beams A and/or A', impinging on the ideal symmetric beam splitter H_1, and is then detected in either beam U_1 or beam L_1, while the other propagates in beams B and/or B' to the ideal symmetric beam splitter H_2 and is detected in either U_2 or L_2. The locution "and/or" is a brief way of referring to a quantum-mechanical superposition, the details of which are specified by the state of $1+2$. For example, the state

$$|\Psi\rangle=\frac{1}{\sqrt{2}}[|A\rangle_1|B\rangle_2+|A'\rangle_1|B'\rangle_2] \tag{1}$$

is entangled, whereas

$$|\Phi\rangle=\frac{1}{\sqrt{2}}(|A\rangle_1+|A'\rangle_1)\frac{1}{\sqrt{2}}(|B\rangle_2+|B'\rangle_2) \tag{2}$$

is a product state, though both involve superpositions. Figure 1 also shows two variable phase shifters ϕ_1 and ϕ_2 placed on beams A and B, respectively.

It is easy to show (see, for instance, Ref. [5]) that if the state of $1+2$ is $|\Psi\rangle$ of Eq. (1), then the probabilities of joint and single detections by the various detectors are

$$P(U_1U_2)=P(L_1L_2)=\tfrac{1}{4}[1-\cos(\phi_1+\phi_2)] \ , \tag{3a}$$

$$P(U_1L_2)=P(L_1U_2)=\tfrac{1}{4}[1+\cos(\phi_1+\phi_2)] \ , \tag{3b}$$

$$P(U_1)=P(L_1)=P(U_2)=P(L_2)=\tfrac{1}{2} \ . \tag{3c}$$

On the other hand, the probabilities of joint and single detections when the state is $|\Phi\rangle$ of Eq. (2) are

$$P(U_1U_2)=P(U_1)P(U_2), \text{ etc.} \tag{4}$$

(for all pairs of outcomes), and

$$P(U_1)=\tfrac{1}{2}(1-\sin\phi_1) \ , \tag{5a}$$

$$P(L_1)=\tfrac{1}{2}(1+\sin\phi_1) \ , \tag{5b}$$

$$P(U_2)=\tfrac{1}{2}(1-\sin\phi_2) \ , \tag{5c}$$

$$P(L_2)=\tfrac{1}{2}(1+\sin\phi_2) \ . \tag{5d}$$

Clearly, Eqs. (3) show that the preparation of $1+2$ in state $|\Psi\rangle$ yields two-particle fringes but no one-particle fringes, where we use the word "fringe" to refer to sinusoidal dependence upon the variable phase angles. The product state $|\Phi\rangle$ produces one-particle fringes but no *genuine* two-particle fringes, since the variation of $P(U_1U_2)$ with ϕ_1 and ϕ_2 is due only to the variation of $P(U_1)$ and $P(U_2)$ separately, as shown in Eq. (4).

Of course, the most striking feature of the foregoing equations is the sinusoidal behavior of the joint probabilities in Eqs. (3a) and (3b) while the singles are flat [Eq. (3c)]. This feature illustrates a natural extension of Feynman's famous rule:

"When an event can occur in several alternative ways, the probability amplitude for the event is the sum of the probability amplitudes for each way considered separately. There is interference. . . . If an experiment is performed which is capable of determining whether one or

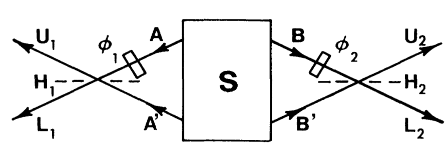

FIG. 1. Schematic two-particle four-beam interferometer using beam splitters H_1 and H_2.

another alternative is actually taken, the probability of the event is the sum of the probabilities for each alternative. The interference is lost." (Ref. [6], p. 37-10).

The second part of Feynman's rule can be naturally extended in the spirit of Einstein, Podolsky, and Rosen [7] by replacing "is performed" by "could be performed without disturbing the system." When $1+2$ is prepared in state $|\Psi\rangle$, one could determine which path particle 1 takes from S to H_1 by placing detectors in the beams B and B', and this determination does not disturb particle 1, hence there is no one-particle interference; likewise for particle 2. Once the pair of particles has gone beyond the beam splitters, however, there is no way to determine whether $1+2$ takes the composite path A,B or the composite path A',B', and hence there is two-particle interference. If $1+2$ is prepared in the product state $|\Phi\rangle$, there is no correlation of the one-particle paths, and hence it is not possible to determine the path of one particle by catching the other; therefore, there is one-particle interference.

States $|\Psi\rangle$ and $|\Phi\rangle$ are extreme cases, the first exhibiting maximum visibility of two-particle interference fringes and minimum visibility of the one-particle fringes, the second exhibiting the opposite. (For the present we use the term "visibility" intuitively, anticipating a precise definition in Sec. II.) These two cases suggest a kind of complementarity between one-particle interference and two-particle interference, as was already noted by Horne and Zeilinger [8]. The purpose of this paper is to formulate this complementarity precisely and to demonstrate it for the most general state of $1+2$ that can be formulated in terms of the single-particle states $|A\rangle_1$, $|A'\rangle_1$, $|B\rangle_2$, and $|B'\rangle_2$.

II. DEFINITION OF "VISIBILITY"

The standard definition of visibility in ordinary interferometry is

$$v=\frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}} . \tag{6}$$

When we deal with a single particle i ($i=1,2$) that has varying probability $P(U_i)$ depending on ϕ_i, it is natural to replace Eq. (6) by

$$v_i=\frac{[P(U_i)]_{\max}-[P(U_i)]_{\min}}{[P(U_i)]_{\max}+[P(U_i)]_{\min}} . \tag{7}$$

Clearly v_i lies in the interval [0,1] and has the minimum value 0 if $P(U_i)$ does not vary with ϕ_i and the maximum value 1 if $P(U_i)$ vanishes for some values of ϕ_i.

It is tempting to write an analogous definition for the visibility of two-particle fringes:

$$v_{12}=\frac{[P(U_1U_2)]_{\max}-[P(U_1U_2)]_{\min}}{[P(U_1U_2)]_{\max}+[P(U_1U_2)]_{\min}} \text{ (No)} . \tag{8}$$

However, this definition fails to capture the intended sense of two-particle visibility because it yields the value $v_{12}=1$ when $1+2$ are prepared in the product state $|\Phi\rangle$, even though the variation of $P(U_1U_2)$ with ϕ_1,ϕ_2 is due [according to Eq. 3(a)] entirely to the variation of $P(U_1)$ with ϕ_1 and of $P(U_2)$ with ϕ_2. Hence one is motivated to define a "corrected" joint probability

$$P'(U_1U_2)=P(U_1U_2)-P(U_1)P(U_2) \tag{9}$$

and replace $P(U_1U_2)$ throughout Eq. (8) by $P'(U_1U_2)$. But the resulting definition for v_{12} is also unsatisfactory because of excessive subtraction. It has the consequence that $[P'(U_1U_2)]_{\min}$ can be negative and the denominator $[P'(U_1U_2)]_{\max}+[P'(U_1U_2)]_{\min}$ can be zero. A better "corrected" joint probability is

$$\bar{P}(U_1U_2)=P(U_1U_2)-P(U_1)P(U_2)+\tfrac{1}{4} . \tag{10}$$

This expression compensates for the excessive subtraction inherent in $P'(U_1U_2)$ by adding $\frac{1}{4}$, which is the value of $P(U_1)P(U_2)$ for all ϕ_1,ϕ_2 when $1+2$ is prepared in the state $|\Psi\rangle$ of Eq. (1). The definition we propose for two-particle visibility is

$$v_{12}=\frac{[\bar{P}(U_1U_2)]_{\max}-[\bar{P}(U_1U_2)]_{\min}}{[\bar{P}(U_1U_2)]_{\max}+[\bar{P}(U_1U_2]_{\min}} . \tag{11}$$

Obviously v_{12} is unity when $1+2$ is prepared in $|\Psi\rangle$ and zero when it is prepared in $|\Phi\rangle$, in agreement with the intuitive considerations of Sec. I. When we examine the most general state definable in terms of $|A\rangle_1$, $|A'\rangle_1$, $|B\rangle_2$, and $|B'\rangle_2$, it will be clear that v_{12} always lies in the interval [0,1].

III. COMPLEMENTARITY IN A SPECIAL FAMILY OF STATES

In order to exhibit the complementarity of one- and two-particle interference without distracting complications, we shall first restrict our attention to the following one-parameter family of cases:

$$|\Psi(\alpha)\rangle=\frac{1}{\sqrt{2}}\cos(\alpha)[|A\rangle_1|B\rangle_2+|A'\rangle_1|B'\rangle_2] + \frac{1}{\sqrt{2}}\sin(\alpha)[|A\rangle_1|B'\rangle_2+|A'\rangle_1|B\rangle_2] . \tag{12}$$

Clearly $\alpha=0$ yields the $|\Psi\rangle$ of Eq. (1) and $\alpha=\pi/4$ yields the $|\Phi\rangle$ of Eq. (2). It is straightforward to show (as in Ref. [5]) that

$$P(U_i)=\frac{1}{2}[1-\sin(2\alpha)\sin\phi_i] \quad (i=1,2) \tag{13}$$

and that

$$\bar{P}(U_1U_2)=\tfrac{1}{4}[1-M\cos(\phi_1)\cos(\phi_2)+N\sin(\phi_1)\sin\phi_2] , \tag{14}$$

where

$$M=\cos(2\alpha) , \quad N=\cos^2(2\alpha) . \tag{15}$$

To find $[\bar{P}(U_1U_2)]_{\max}$ and $[\bar{P}(U_1U_2)]_{\min}$, we set derivatives to zero:

$$0=\frac{\partial \bar{P}(U_1U_2)}{\partial\phi_2}\Longrightarrow \cos(\phi_1)\sin\phi_2=\frac{-N}{M}\sin(\phi_1)\cos\phi_2\ , \tag{16a}$$

$$0=\frac{\partial \bar{P}(U_1U_2)}{\partial\phi_1}\Longrightarrow \sin(\phi_1)\cos\phi_2=\frac{-N}{M}\cos(\phi_1)\sin\phi_2\ . \tag{16b}$$

If $N=\pm M$, then $\cos 2\alpha=\pm 1$ and $\alpha=0$ or $\pi/2$, in which case we recover the $|\Psi\rangle$ of Eq. (1) or the similar state with $|B\rangle$ and $|B'\rangle$ interchanged. In either case $v_i=0$ and $v_{12}=1$. If $N\neq M$, then $\bar{P}(U_1U_2)$ is stationary only when $\cos(\phi_1)\cos\phi_2$ and $\sin(\phi_1)\sin\phi_2$ have the following pairs of values:

$$(\pm 1,0)\Longrightarrow \bar{P}(U_1U_2)=\tfrac{1}{4}(1\mp M)\ , \tag{17a}$$

$$(0,\pm 1)\Longrightarrow \bar{P}(U_1U_2)=\tfrac{1}{4}(1\pm N)\ . \tag{17b}$$

Since $|M|$ is never less than $|N|$ by Eqs. (15), it follows that

$$[\bar{P}(U_1U_2)]_{\max}=\tfrac{1}{4}(1+|M|)\ , \tag{18a}$$

$$[\bar{P}(U_1U_2)]_{\min}=\tfrac{1}{4}(1-|M|)\ , \tag{18b}$$

Hence by Eq. (11),

$$v_{12}=|\cos(2\alpha)|\ . \tag{19a}$$

But from Eqs. (6) and (12)

$$v_1=v_2=|\sin(2\alpha)| \tag{19b}$$

Consequently, in all cases

$$v_i^2+v_{12}^2=1\quad (i=1,2)\ . \tag{20}$$

As expected, the increase in visibility of the two-particle fringes is compensated by the decrease in visibility of the one-particle fringes, and conversely. The extreme cases of the states $|\Psi\rangle$ and $|\Phi\rangle$, discussed previously, are instances of the complementarity equation (20).

It must be emphasized, however, that in the general case (to be discussed in Sec. IV) $v_i^2+v_{12}^2$ only has unity as an upper bound.

IV. COMPLEMENTARITY IN THE GENERAL CASE

The most general state that can be formed from $|A\rangle_1$, $|A'\rangle_1$, $|B\rangle_2$, and $|B'\rangle_2$ is

$$\begin{aligned}|\Theta\rangle=\cos(\alpha)[&\cos(\beta)|A\rangle_1|B\rangle_2\\&+e^{i\lambda}\sin(\beta)|A'\rangle_1|B'\rangle_2]\\&+\sin(\alpha)[e^{i\mu}\cos(\gamma)|A\rangle_1|B'\rangle_2\\&+e^{i\nu}\sin(\gamma)|A'\rangle_1|B\rangle_2]\ ,\end{aligned} \tag{21}$$

since any overall phase can be omitted. We wish to obtain expressions for the visibilities v_i and v_{12} as functions of the parameters α, β, γ, λ, μ, and ν appearing in $|\Theta\rangle$. It is evident, however, that the visibilities are independent of λ, μ, and ν, because the basis change (which could be accomplished by inserting phase shifters)

$$|A\rangle_1=e^{i\rho}|\bar{A}\rangle_1\ , \tag{22a}$$

$$|A'\rangle_1=e^{i\rho'}|\bar{A}'\rangle_1\ , \tag{22b}$$

$$|B\rangle_2=e^{i\sigma}|\bar{B}\rangle_2\ , \tag{22c}$$

$$|B'\rangle_2=e^{i\sigma'}|\bar{B}'\rangle_2\ , \tag{22d}$$

with properly chosen ρ, ρ', σ, and σ', yields an expression for $|\Theta\rangle$ with all real coefficients. If we then drop the bars from $|\bar{A}\rangle_1$, $|\bar{A}'\rangle_1$, $|\bar{B}\rangle_2$, and $|\bar{B}'\rangle_2$ for notational convenience, we obtain as the most general state for our purposes

$$\begin{aligned}|\Theta(\alpha,\beta,\gamma)\rangle=\cos(\alpha)[&\cos(\beta)|A\rangle_1|B\rangle_2\\&+\sin(\beta)|A'\rangle_1|B'\rangle_2]\\&+\sin(\alpha)[\cos(\gamma)|A\rangle_1|B'\rangle_2\\&+\sin(\gamma)|A'\rangle_2|B\rangle_2]\ .\end{aligned} \tag{23}$$

Another way to put the matter is that the phases λ, μ, and ν of Eq. (21) only have the effect of shifting the values of ϕ_1 and ϕ_2 that maximize or minimize $P(U_1)$,$P(U_2)$ and $P(U_1,U_2)$, and these shifts do not affect the visibilities.

Straightforward calculations using $|\Theta(\alpha,\beta,\gamma)\rangle$ yield

$$P(U_1)=\tfrac{1}{2}\{1-\sin(2\alpha)[\sin(\beta)\cos\gamma+\cos(\beta)\sin\gamma]\sin\phi_1\}\ , \tag{24a}$$

$$P(U_2)=\tfrac{1}{2}\{1-\sin(2\alpha)[\cos(\beta)\cos\gamma+\sin(\beta)\sin\gamma]\sin\phi_2\}\ , \tag{24b}$$

and

$$\bar{P}(U_1U_2)=\tfrac{1}{4}[1-M'\cos(\phi_1)\cos\phi_2+N'\sin(\phi_1)\sin\phi_2]\ , \tag{25}$$

where

$$M'=\cos^2(\alpha)\sin(2\beta)-\sin^2(\alpha)\sin(2\gamma)\ , \tag{26a}$$

$$\begin{aligned}N'&=\cos^2(\alpha)\sin(2\beta)+\sin^2(\alpha)\sin(2\gamma)\\&\quad-2\sin^2(\alpha)\cos^2(\alpha)[\sin(2\beta)+\sin(2\gamma)]\\&=M'\cos(2\alpha)\ .\end{aligned} \tag{26b}$$

Since $\bar{P}(U_1U_2)$ has exactly the same dependence upon ϕ_1 and ϕ_2 as in Eq. (13) of Sec. III, we obtain results corresponding to Eqs. (16a) and (16b) for the stationary values of $\bar{P}(U_1U_2)$. If $N'\neq M'$ then

$$\bar{P}(U_1U_2)=\tfrac{1}{4}(1\mp M')\ , \tag{27a}$$

$$\bar{P}(U_1U_2)=\tfrac{1}{4}(1\pm N')\ . \tag{27b}$$

By Eq. (26b), however, $|M'|\geq|N'|$. Hence

$$[\bar{P}(U_1U_2)]_{\max}=\tfrac{1}{4}(1+|M'|)\ , \tag{28a}$$

$$[\bar{P}(U_1U_2)]_{\min}=\tfrac{1}{4}(1-|M'|)\ , \tag{28b}$$

$$v_1^2=\frac{1}{2}\sin^2(2\alpha)[1+\sin(2\beta)\sin(2\gamma)-\cos(2\beta)\cos(2\gamma)] \ , \tag{29a}$$

$$v_2^2=\frac{1}{2}\sin^2(2\alpha)[1+\sin(2\beta)\sin(2\gamma)+\cos(2\beta)\cos(2\gamma)] \ , \tag{29b}$$

$$v_{12}^2=\cos^4(\alpha)\sin^2(2\beta) -2\sin^2(\alpha)\cos^2(\alpha)\sin(2\beta)\sin(2\gamma) +\sin^4(\alpha)\sin^2(2\gamma) \ , \tag{29c}$$

and

$$v_i^2+v_{12}^2=1-[\sin^2(\alpha)\cos(2\gamma)-(-1)^i\cos^2(\alpha)\cos(2\beta)]^2 \quad (i=1,2) \ . \tag{30}$$

By inspection,

$$v_i^2+v_{12}^2\le 1 \ , \tag{31}$$

or equivalently (because visibilities are non-negative)

$$0\le v_i v_{12}\le \tfrac{1}{2} \ . \tag{32}$$

If $N'=\pm M'$ then $\alpha=0$ or $\pi/2$, and the same results (31) and (32) are obtained by separate calculations. Inequalities (31) and (32) are the desired expressions for the complementarity of one- and two-particle fringes.

V. EXTENSIONS

Most two-particle experiments have been carried out more or less in the framework sketched in this paper, with each particle impinging on (both sides of) a beam splitter having two exit channels. However, the first two-particle experiment—that of Ghosh and Mandel [9]—placed small detectors in the region of overlap of beams A and A', and of B and B', respectively, and continuously scanned the two-particle radiation. The positions x_1 and x_2 of the detectors along a single axis took the place of the phase angles ϕ_1 and ϕ_2 our schematic arrangement. Thus their experimental arrangement permitted a continuum of outcomes for each particle, rather than only two. This is also the case in Ref. [10]. Nevertheless, with small modifications, the concepts of one- and two-particle visibilities of this paper can be extended to the situation of continuous outcomes. One need only replace the corrected joint probability $\overline{P}(U_1U_2)$ of Eq. (9) by a corrected joint probability density

$$\overline{\rho}(x_1,x_2)=\rho(x_1,x_2)-\rho(x_1)\rho(x_2)+\left[\frac{K}{2\pi}\right]^2 . \tag{33}$$

Here $\rho(x_i)$ is the probability density of catching particle i at x_i $(i=1,2)$, $\rho(x_1,x_2)$ is the probability density of catching the two particles at x_1 and x_2, respectively, and

$$K=\frac{2\pi}{\lambda}\sin\left[\frac{\theta}{2}\right] . \tag{34}$$

λ being the wavelength of the radiation and θ the crossing angles of the beams. The term $(K/2\pi)^2$ plays the role of $\frac{1}{4}$ of Eq. (9), and the single- and joint-probability densities have been normalized to one cycle of the fringe pattern. Visibility v_{12} can be defined in terms of ρ analogously to Eq. (10). The inequalities (31) and (32) are valid.

The experiment proposed by Franson [11] and carried out by Ou *et al.* [12] and Kwiat *et al.* [13] falls roughly under the framework of this paper, since it works with fixed detectors. We have not carried out a detailed calculation of complementarity for the Franson arrangement, but it would be worthwhile to do so, because that arrangement permits an easy scanning from the case of high one-particle and low two-particle visibility to the opposite case [14]. We note that the foregoing complementarity can be generalized to systems of three or more particles, but the expressions are more complex.

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APPENDIX

We stress that the general state $|\Theta\rangle$ of Eq. (21) can be prepared. First, we modify the arrangement of Fig. 1, as shown in Fig. 2. The detectors and phase shifters of Fig. 1 are omitted. The beam splitters are retained but are not assumed to be symmetrical, so that their reflectivity r and transmittivity t are any positive real numbers such that $r^2+t^2=1$.

There is no need for our purpose to make the r's (and hence the t's) of H_1 and H_2 different. We label the beams emerging from H_1 and H_2 with the same letters as the incident beams in the same direction. Finally, an absorber and a phase shifter are inserted in each emerging beam. The absorber in A multiplies the amplitude in that beam by a $(0\le a\le 1)$, and the phase shift is ρ; in A' the parameters are a' and ρ', in B they are b and σ, and in B' they are b' and σ'.

If the column vector $\binom{c}{c'}$ represents $c|A\rangle+c'|A'\rangle$ when we are concerned with particle 1 and $c|B\rangle+c'|B'\rangle$ when we are concerned with particle 2, then the matrix representing the transformation T_i of the initial state of particle i is

$$T_1=\begin{bmatrix} ae^{i\rho} & 0 \\ 0 & a'e^{i\rho'}\end{bmatrix}\begin{bmatrix} t & ir \\ ir & t\end{bmatrix} =\begin{bmatrix} ae^{i\rho}t & ae^{i\rho}ir \\ a'e^{i\rho'}ir & a'e^{i\rho'}t\end{bmatrix} , \tag{A1}$$

$$T_2=\begin{bmatrix} be^{i\sigma} & 0 \\ 0 & b'e^{i\sigma'}\end{bmatrix}\begin{bmatrix} t & ir \\ ir & t\end{bmatrix} =\begin{bmatrix} be^{i\sigma}t & be^{i\sigma}ir \\ b'e^{i\sigma'}ir & b'e^{i\sigma'}t\end{bmatrix} , \tag{A2}$$

and the direct product

$$T = T_1 \otimes T_2 \tag{A3}$$

represents the transformation effected by the entire apparatus on the initial state of 1+2.

If the initial state emerging from the source is $|\Psi\rangle$ of Eq. (1), its representation by column vectors (if the normalization factor $2^{-1/2}$ is suppressed) is

$$|\Psi\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}_1 \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}_2 + \begin{bmatrix} 0 \\ 1 \end{bmatrix}_1 \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}_2 . \tag{A4}$$

The final state (not normalized) is

$$\begin{aligned} |\Psi_f\rangle = T|\Psi_i\rangle &= T_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}_1 \otimes T_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix}_2 \\ &\quad + T_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_1 \otimes T_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_2 \\ &= \begin{bmatrix} ae^{i\rho}t \\ a'e^{i\rho'}ir \end{bmatrix}_1 \otimes \begin{bmatrix} be^{i\sigma}t \\ b'e^{i\sigma'}ir \end{bmatrix}_2 \\ &\quad + \begin{bmatrix} ae^{i\rho}ir \\ a'e^{i\rho'}t \end{bmatrix}_1 \otimes \begin{bmatrix} be^{i\sigma}ir \\ b'e^{i\sigma'}t \end{bmatrix}_2 . \end{aligned} \tag{A5}$$

Rewriting explicitly in terms of $|A\rangle_1$, $|A'\rangle_1$, $|B\rangle_2$, and $|B'\rangle_2$ yields

$$\begin{aligned} |\Psi_f\rangle &= (abe^{i(\rho+\sigma)})(t^2 - r^2)|A\rangle_1|B\rangle_2 \\ &\quad + (ab'e^{i[\rho+\sigma'+(\pi/2)]})2rt|A\rangle_1|B'\rangle_2 \\ &\quad + (a'be^{i[\rho'+\sigma+(\pi/2)]})2rt|A'\rangle_1|B\rangle_2 \\ &\quad + (a'b'e^{i(\rho'+\sigma')})(t^2 - r^2)|A'\rangle_1|B'\rangle_2 . \end{aligned} \tag{A6}$$

It is easily checked that the proper choice of t, a, a', b, b', ρ, ρ', σ, and σ', will yield $|\Psi_f\rangle$, in agreement with $|\Theta\rangle$ of Eq. (21), except for normalization.

Note that the foregoing argument does not depend upon taking the initial state to be $|\Psi_i\rangle$ of Eq. (14). Any *entangled* state will suffice to generate the general state $|\Theta\rangle$ of Eq. (21), but no *product* state will suffice. Finally, in order to do interferometry with the general state $|\Theta\rangle$, one inserts the apparatus of Fig. 2 as the source S of Fig. 1.

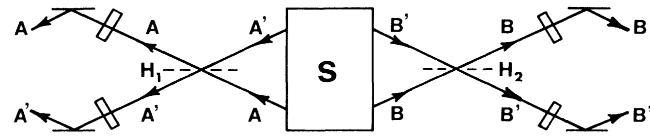

FIG. 2. Production of a general state. Each box beyond the beam splitters represents both an absorber and a phase shifter.

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