Math 876: PDE Seminar Spring 2022

Course Info

Instructor: Jonathan Jaquette

Times: Tue, Thu 9:30am 10:45am

Location: CGS 111A (871 Commonwealth Avenue, Boston MA)

Office Hours: To be decided on the first day of class

Textbook: Sell, G. R., & You, Y. (2002). *Dynamics of Evolutionary Equations*. Springer Science & Business Media. (https://link.springer.com/book/10.1007/978-1-4757-5037-9)

Course Website: Apart from the textbook, all course readings, announcements, and supplemental materials will be posted through the course Blackboard page.

Course Content

The central focus in this class is to study the dynamics of evolutionary equations. In particular we will study the qualitative behavior of solutions of partial differential equations which evolve in time. Since the nonlinearities in such equations are often nontrivial, a good dynamical theory of these infinite dimensional systems is essential.

Course Objectives

By the end of the course, students will be able to:

- Given a nonlinear evolutionary equation, analyze its global dynamics and qualitative features.
- Critically read the textbook and journal articles, summarize the major points, and discuss the finer details.
- Prepare and deliver 10-20 minute presentations on PDEs.

Course Pre-Requisites:

The standard pre-requisite for this class is to have taken (or be currently taking) *MA 776 Partial Differential Equations*. Alternatively, if you have taken some number of the classes below, then you are probably well prepared for MA 876.

- MA 711 Real Analysis
- MA 717 Functional Analysis
- MA 771 Introduction to Dynamical Systems
- MA 775 Ordinary Differential Equations

If you have any questions or concerns about your preparation for this course, come talk to me about it!

Seminar Format

This course is a seminar and, unlike a traditional course, almost all of the talking in our meetings is done by the students. The main work of the seminar meeting will be presentations on selected topics, discussion of the readings and presentations, discussion of student generated questions, and presentation and discussion of homework problems. I'd like the seminar atmosphere to be lively but not intimidating. The goal of the group should be to push everyone's understanding. If something isn't clear, we need to stop and clarify.

In this format, the seminar members will be expected to have mastered all the basic background material before seminar, and the seminar meeting will be devoted to reinforcing, extending and enhancing your knowledge of the seminar topics. Students presenting material in any given week must thoroughly prepare those presentations in advance. Students not presenting material should still study all of the material to be presented in advance so that they may ask good questions and participate fully in the discussions. If you are unable to attend class for any reason, please contact me as soon as possible.

Many of us are probably new to the seminar format. Please feel free to make suggestions of things we can do differently that will help you learn better along the way.

Meeting Organization

Below is an example agenda for one week's seminar:

Tuesday Class:

- Summary of the week's reading (10-15 min)
- Student presentation(s) (20-30 min)
- Discussion of students' prepared questions on the reading (30-40 min)

Thursday Class:

- Student presentation(s) (20-30 min)
- Discussion of homework problems (40-50 min)
- Closing, and planning for the next week (5-10 min)

Discussion Questions:

Each week there will be about 25 pages from the book that everyone should read by Tuesday.

At least 1 hour before class on Tuesday, each student must email me a list of **at least 3 questions** they had from the reading. My suggestion would be to compile your list of questions as you are reading the week's material.

In class on Tuesday we will go around the room. Each person will pose one of their questions and the group will discuss. If there is time remaining we will discuss any further questions/topics the group finds to be of interest.

Presentations

Many seminars will include a short presentation or two prepared by students. I'll ask for volunteers for presentations the week before. Presentations may include discussing a section from our text or another book, readings from journals, examples relating to the week's reading, or numerical demonstrations.

Each presentation should be 10-20 minutes and leave 5-10 minutes for questions. Depending on the size of the class, each student may expect on average to give a presentation every other week.

In weeks that you are giving a presentation, you should talk to me in office hours before hand to discuss your talk outline/ask questions about the material/etc.

Homework Problems

Each week there will be a handful of homework problems to accompany the reading. You are strongly encouraged to work together on problems. While you will not be turning in a write up for these problems, you are expected to attempt as many problems as you can and come to class prepared to discuss them.

At the beginning of each Thursday seminar we'll decide who will lead the discussion on which homework problems. You'll let me know the problems you feel most comfortable with and I'll respect your preferences. Difficult problems we can all work on together.

Tentative Schedule of Material

Week	Tues	Thurs	Topic	Reading
0		1/20	Introduction	
1	1/25	1/27	Spectral Methods for PDEs	Ch 1 & Uecker (2009) §1-3,A
2	2/1	2/3	Dynamical Systems: Basic Theory	Ch 2.1 - 2.3.4
3	2/8	2/10	Attractors and Linear Semigroups	Ch 2.3, Ch 3.1 - 3.5
4	2/15	2/17	Analytic Semigroups	Ch 3.6, 3.7
5		2/24	Examples of Semigroups	Ch 3.8
6	3/1	3/3	Variation of Constants Formula	Ch 4.1 – 4.2.2
Spring Break				
7	3/15	3/17	Linear Semi-flows	Ch 4.2.3 – 4.4
8	3/22	3/24	Nonlinear Semi-flows	Ch 4.6, 4.7
9	3/29	3/31	Some Nonlinear PDEs	Ch 5.1, 5.4
10	4/5	4/7	Exponential Dichotomies	Ch 4.5
11	4/12	4/14	Major Features of Dynamical Systems	Ch 7.1, 7.2
12	4/19	4/21	Open Topic 1	TBD
13	4/26	4/28	Open Topic 2	TBD
14	5/3		Open Topic 3	TBD

Numerics

To supplement the theoretical side of things we will cover in class, you will have the opportunity to use numerics to study solutions of nonlinear PDEs. MATLAB and Mathematica are my preferred mathematical programming languages, but you can use whichever computer program(s) you feel most comfortable with.

Open Topics

I have left the topics for the last three weeks open for the class to decide on. For these weeks I propose we move away from the text book and read seminal research papers on dynamics of PDEs. Some possible topics could include: bifurcations, chaos, computer assisted proofs, inertial manifolds, and/or other types of evolutionary equations (eg. delay differential equations, integro-differential equations). I am very open to other suggestions you might have. Around spring break, I will come up with a list of possible papers and we will finalize the topics for the last three weeks.

Course Grades

The grade will be A for those students who remain involved with the course throughout the semester and contribute to its success.

Acknowledgements

Thank you to Jennifer Balakrishnan and my professors in the math & physics departments at Swarthmore College for being an inspiration in designing this course.