Math 876: PDE Seminar 2022
Week 14: Global dynamics of gradient systems

May 3, 2022

General: Global attractors are important (when they exist) because they attract all of the solutions that exist globally in time. But an attractor could still be a ridiculously complicated, have fractal or infinite dimension, etc. Proving that an attractor exists is the end of the story, but rather the beginning of trying to figure out what's going on inside. Morse decompositions give us a road map for how trajectories traverse the global attractor.

Primary Reading: First read "How to Read a Research Paper" by Matt Baker (Notices of the AMS). Then do a "Speed Read" of all three papers below (posted on Blackboard).

(A) Mischaikow, K. (1995). Global asymptotic dynamics of gradient-like bistable equations. *SIAM journal on mathematical analysis*, 26(5), 1199-1224.

(B) Arioli, G., & Koch, H. (2010). Computer-assisted methods for the study of stationary solutions in dissipative systems, applied to the Kuramoto–Sivashinski equation. *Archive for rational mechanics and analysis*, 197(3), 1033-1051.

(C) Wilczak, D., & Zgliczyński, P. (2020). A geometric method for infinite-dimensional chaos: Symbolic dynamics for the Kuramoto-Sivashinsky PDE on the line. *Journal of Differential Equations*, 269(10), 8509-8548.

Next, do a more in depth reading of either Paper A (pages 1-7), or Paper B (pages 1-6).

Secondary Reading: You might go back and read §7.2 on dynamics of gradient systems again.

Reading Questions: Email me at least 3 questions on the reading at least an hour before class on Tuesday.

Presentations:

Gradient Systems and Morse Decompositions

Tell us about gradient systems and Morse decompositions from §7.2. In particular, explain what is going on in Figures 7.3 and and 7.4.

Global asymptotic dynamics of gradient-like bistable equations

Tell us about the paper "Global asymptotic dynamics of gradient-like bistable equations". What do the dynamics for this model ODE in (2)-(3) look like? How does this relate to Figures 1-2? What does Theorem 1.2 tell us? Can you describe (H1), (H2), (H3) in **short** bullet points.

The Kuramoto-Sivashinsky equation

Tell us about the paper "Computer-assisted methods for the study of stationary solutions in dissipative systems, applied to the Kuramoto–Sivashinski equation". Explain some (but not necessarily all) of the bifurcation diagram in Figure 1. What are some of the take aways from Figure 3?