§V Morris-Lecar model (ML) - 1981

- giant muscle fiber of Pacific giant barnacle
- HH-style model, but second order

§V(a) The model

The ML-model includes three current channels. The first is a passive leak channel. The second is a voltage gated potassium channel with a single activating gate, represented by the gating variable $w$. The third is a voltage gated calcium channel with a single activating gate; it is assumed that this channel reacts quickly to changes in the membrane potential $V$ so that it is determined by the function $m_\infty(V)$ rather than a differential equation.

\[
C \frac{dV}{dt} = I_{\text{input}}(t) - \bar{g}_{Ca} m_\infty(V)(V - V_{Ca}) - \bar{g}_K w(V - V_K) - \bar{g}_L (V - V_L)
\]

\[
\frac{dw}{dt} =
\]

Typical values of the parameters are:

- Nernst potentials: $V_{Ca} = +120 \text{ mV}$, $V_K = -84 \text{ mV}$, $V_L = -60 \text{ mV}$
- maximum conductances: $\bar{g}_{Ca} = 4.4 \mu \text{mho}$, $\bar{g}_K = 8 \mu \text{mho}$, $\bar{g}_L = 2 \mu \text{mho}$

and $C = 1 \text{ nF}$. A typical choice for the nonlinear functions $m_\infty(V)$, $w_\infty(V)$, and $\tau_w(V)$, based in part on experimental measurements, is:

\[
m_\infty(V) = \frac{1}{2} \left(1 + \tanh \frac{V - V_1}{V_2}\right), \quad w_\infty(V) = \frac{1}{2} \left(1 + \tanh \frac{V - V_3}{V_4}\right), \quad \tau_w(V) = \tau_0 \sech \frac{V - V_3}{2V_4},
\]

where

\[
V_1 = -1.2 \text{ mV}, \quad V_2 = 18 \text{ mV}, \quad V_3 = 2 \text{ mV}, \quad V_4 = 30 \text{ mV}, \quad \tau_0 = 25 \text{ ms}.
\]

The shape of these functions is what matters:
§V(b)  dynamics of an AP in the ML-model

Simulation of an AP in the ML-model
Let's look at this AP in the \((V, w)\) phase plane:

- recall how to handle pulsed inputs, by piecing together the behavior from several autonomous systems
- \(V\)-nullcline: \(\frac{dV}{dt} = 0\) so

\[
\begin{align*}
V & = -60 \text{ mV} \\
& = 0 \\
& = 40 \text{ mV}
\end{align*}
\]

- \(w\)-nullcline: \(\frac{dw}{dt} = 0\) so

\[
\begin{align*}
w & = -60 \text{ mV} \\
& = 0 \\
& = 40 \text{ mV}
\end{align*}
\]
even Hodgkin and Huxley did not really know why their model produced spikes.

ML-model suggests how to construct a 2D approximation to the HH-model of a squid giant axon

- Na activation ($m$):
  \[ \tau_m(V) \text{ is always small,} \]

- K activation ($n$) and Na inactivation ($h$):
  \[ \tau_n(V) \approx \tau_h(V) \]
  \[ n_\infty(V) \approx 1 - h_\infty(V) \]

→ this approximation to the original HH-model is two dimensional
  the phase plane looks a lot like the ML-model, and a lot like FHN

**§V(d)  FHN redux**

*FitzHugh was brilliant.*
§VI Ermentrout-Kopell model (1986)

- also called $\theta$-model or theta-model
- one dimensional, but mathematically nicer than LIF
- very abstract
- mathematical simplicity $\rightarrow$ useful to develop intuition about large networks

§VI(a) motivation

Recall why 1d models don’t make AP:

- Want this $V(t)$ as a solution:

- look at the trajectory of an AP in 1D phase space

- so 1st order systems can not make an AP spike unless we include threshold / reset by hand

- note:
Recall how AP looks in FHN, the simplest spiking model

- excitable, $I = 0$ phase space pic

- allows $dV/dt > 0$ and $dV/dt > 0$ at same $V$, because of different values of recovery variable $w$

- phase space is 2D, but don’t ‘feel’ all of it - spend most time on a loop
§VI(b) the $\theta$-neuron model

\[
\frac{d\theta}{dt} = f(\theta) \quad \text{where} \quad \bullet \ f(\theta) = (1 - \cos \theta) + (1 + \cos \theta) a \\
\bullet \ -\pi \leq \theta < \pi \\
\bullet \ \theta = \pm \pi \text{ counts as ‘firing’}
\]

rough description:

What does $f(\theta)$ look like?

[Diagram of $f(\theta)$ function with points at $\pm \pi$]
§VI(c) Analysis and dynamics

- phase space =

- expect different dynamics in $a < 0$ and $a > 0$ (why?)

**case $a < 0$**

- $f(\theta)$ has two zeros, at $\theta_{fp} = \cos^{-1}\left(\frac{1 + a}{1 - a}\right)$

- phase space sketch:

  ![Phase space sketch](image)

- f.p. in $\theta < 0$ is stable
- f.p. in $\theta > 0$ is unstable

- time-series:
case $a > 0$

- $f(\theta)$ has no fixed points

- phase space sketch:

- no fixed points

- time-series:

The $\theta$-model is mathematically simple, and the behavior of a $\theta$-neuron intuitive. It has many uses, such as building intuition of networks of interacting neurons. We will use it to demonstrate network phenomena in a simple model, which we will then recognize in other more complicated and realistic models.