ERRATA FOR "COMPUTING LOCAL p-ADIC HEIGHTS ON HYPERELLIPTIC CURVES"

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The main results of the paper [BB12], giving an algorithm for a Coleman integral of a differential with poles in non-Weierstrass disks (Algorithm 4.8) and an algorithm for the Coleman-Gross local height at p for a hyperelliptic curve (Algorithm 5.8) are unchanged. However, along the way, a number of sign changes are needed to ensure the definitions are consistent. (Fortunately, the parity of sign swaps relevant in the two algorithms is even.)

We are very grateful to Stevan Gajovic for bringing a number of these corrections and clarifications to our attention.

(1) Definition 2.2: Third kind differentials have at most simple poles.
(2) Definition 2.4: The global symbol is a sum of local symbols $\langle \omega, \rho \rangle_A$, where

$$\langle \omega, \rho \rangle_A = -\operatorname{Res}_A \left(\omega \int \rho \right).$$

(3) The definition in (13) in Section 4.3 should be that when $\operatorname{Res}_A \omega = 0$, that

$$\langle \omega, \rho \rangle_A = \operatorname{Res}_A \left(\rho \int \omega \right).$$

(4) Algorithm 4.8 uses a slightly more general result than stated in Proposition 2.5: it uses the full force of [Bes00, Proposition 4.10]. The result in [Bes00, Proposition 4.10] is that

$$\langle \omega, \rho \rangle = \Psi(\omega) \cup \Psi(\rho).$$

(5) In Remark 4.9, there is an index missing in the displayed equation immediately after (15): it should read

$$\Psi(\alpha) \cup \Psi(\beta) = \sum_{A \in \mathcal{S} \cup \{R, S\}} \langle \alpha, \beta \rangle_A.$$

(6) In Section 5.2, there are two sign errors. In Proposition 5.12, the sign of the RHS should be changed, after the change of sign in Definition 2.4:

$$\begin{aligned}
\langle \omega, \omega_i \rangle &= \sum_{P \in \operatorname{Res}(\omega) = D} \langle \omega, \omega_i \rangle_P + \langle \omega, \omega_i \rangle_\infty \\
&= -\sum_{P \in D} \operatorname{Res}_P \left(\omega \int \omega_i \right) - \operatorname{Res}_\infty \left(\omega \int \omega_i \right) \\
&= -\int_D \omega_i - \operatorname{Res}_\infty \left(\omega \int \omega_i \right).
\end{aligned}$$

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Moreover, when we compute $\Psi(\omega)$, we use the inverse of the transpose of the cup product matrix N, so the first displayed formula on p. 2429 should instead read

$$\Psi(\omega) = (N^t)^{-1} \begin{pmatrix} \langle \omega, \omega_0 \rangle \\ \vdots \\ \langle \omega, \omega_{2g-1} \rangle \end{pmatrix} = -N^{-1} \begin{pmatrix} \langle \omega, \omega_0 \rangle \\ \vdots \\ \langle \omega, \omega_{2g-1} \rangle \end{pmatrix}.$$

(These are the two sign swaps that cancel out in Algorithm 4.8.)

(7) In Definition 5.9, the residue is taken at the only possible pole, ∞, so we have

$$[\omega_{i-1}] \cup [\omega_{j-1}] = \mathrm{Res}_\infty \left(\omega_{j-1} \int \omega_{i-1} \right).$$

(8) Fixed statement of Proposition 6.5:
Let α be as above and let A be a finite Weierstrass point not equal to $(0,0)$. Let $(x(t), y(t))$ represent the local coordiantes the case of a Weierstrass point $A \neq (0,0)$ we compute the local coordinate $(x(t), y(t))$ at A. Then to compute $\mathrm{Res}_A(\alpha \int \beta)$ with n digits of p-adic precision, we compute $x(t), y(t)$ to $t^{2pn-p-1}$.

References

[BB12] J. S. Balakrishnan and A. Besser, *Computing local p-adic height pairings on hyperelliptic curves*, IMRN **2012** (2012), no. 11, 2405–2444.

[Bes00] A. Besser, *Syntomic regulators and p-adic integration. II. K_2 of curves*, Proceedings of the Conference on p-adic Aspects of the Theory of Automorphic Representations (Jerusalem, 1998), vol. 120, 2000, pp. 335–359.

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