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# What is on today



# 1 Introduction

- Please refer to the syllabus for all course details.
- How lectures will work:
  - Notes will be available online (http://math.bu.edu/people/jbala/123.html) for use in each class.
  - We will use Learning Catalytics (through MyMathLab).
  - Examples will usually be even-numbered exercises from the textbook.
- Chapter 1 of the textbook gives a review of functions (definitions of various functions: polynomials, rational, exponential, logarithmic, trigonometric; transformations of functions, inverse functions, etc.), all of which will play a crucial role throughout our semester.
  - It is very important to be familiar with the content of this foundational first chapter. Please take a look and do some practice problems this week if any of the concepts are unfamiliar or a bit hazy after the summer.
  - You will have a review worksheet covering some of the topics in Chapter 1 during the first discussion session.
  - Let your TF know if you have not covered a specific topic.
  - We will also review some of the more challenging topics (e.g., inverse trig functions) in lecture when they show up.
- Calculus I: Limits, Differentiation and Integration (Chapters 2 to 5 of the textbook).

# 2 The idea of limits: average/instantaneous velocity

Briggs-Cochran-Gillett §2.1 pp. 54–60

## 2.1 Motivating example: average velocity

**2016 Summer Olympics 100m men's race: final results**

| | Distance | Time |
|---|---|---|
| Bolt | 100m | 9.81s |
| Gatlin | 100m | 9.89s |

How fast did they go?

**Recall:** If $s(t)$ is the position at time $t$, the *average velocity* between $t_0$ and $t_1$ is

$$\boxed{v_{av} = \frac{s(t_1) - s(t_0)}{t_1 - t_0}}.$$

The race: https://www.youtube.com/watch?v=4gUW1JikaxQ&feature=youtu.be&t=190

Graphs of position at various times:

Gatlin clearly started faster (covered more distance in the same amount of time), but then Bolt caught up. Their velocities changed during the race.

## 2.2 From average to instantaneous velocity, geometric idea

**Example 1** (§2.1 **Ex. 10**) *The position of an object moving vertically along a line is given by the function* $s(t) = -4.9t^2 + 30t + 20$. *Find the average velocity of the object in the intervals* $[0, 3]$, $[0, 2]$, $[0, 1]$ *and* $[0, h]$ *where* $h > 0$ *is a real number.*

**Example 2** (§2.1 **Ex. 28**) *Let $f(x) = x^3 - x$. Make a table of slopes of secant lines and make a conjecture about the slope of the tangent line at $x = 1$. (Recall that a* ***secant line*** *is a straight line joining two points on a curve.)*

To summarize:

| | | |
|---|---|---|
| The instantaneous velocity at $t = t_0$ | = | "Limit as $t \to t_0$ of the average velocities in the intervals $[t_0, t]$" |

This geometrically corresponds to the following:

| | | |
|---|---|---|
| The slope of the tangent line to $s(t)$ at $(t_0, s(t_0))$ | = | "Limit as $t \to t_0$ of the slopes of the secant lines between $(t_0, s(t_0))$ and $(t, s(t))$" |