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What is on today

1 Definition of limits

Briggs-Cochran-Gillett § 2.2, pp. 61–68

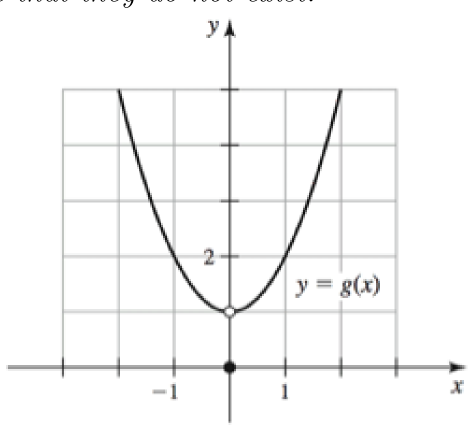
Definition 1 (Limit of a function (Preliminary)) *Suppose the function f is defined for all x near a except possibly at a. If $f(x)$ is arbitrarily close to L (as close to L as we like) for all x sufficiently close (but not equal) to a, we write*

$$\lim_{x\to a} f(x) = L$$

and say the limit of $f(x)$ as x approaches a equals L.

1.1 Finding limits with graphs

Example 2 (§2.2 Ex. 8) *Use the graph of g in the figure to find the following values or state that they do not exist.*



(a) $g(0)$

(b) $\lim_{x\to 0} g(x)$

(c) $g(1)$

(d) $\lim_{x\to 1} g(x)$

1.2 Finding limits with tables

Example 3 (§2.2 **Ex. 12)** *Let* $f(x) = \frac{x^3-1}{x-1}$.

(a) Calculate $f(x)$ *for each value of* x *in the following table.*

x	0.9	0.99	0.999	0.9999
$f(x) = \dfrac{x^3-1}{x-1}$				
x	1.1	1.01	1.001	1.0001
$f(x) = \dfrac{x^3-1}{x-1}$				

(b) Make a conjecture about the value of $\lim\limits_{x\to 1} \dfrac{x^3-1}{x-1}$.

1.3 One-sided limits

Definition 4 (One-sided Limits: a right-sided limit or a left-sided limit)
Right-sided limit*: Suppose* f *is defined for all* x *near* a *with* $x > a$*. If* $f(x)$ *is arbitrarily close to* L *for all* x *sufficiently close to* a *with* $x > a$*, we write*

$$\lim_{x\to a^+} f(x) = L$$

and say the limit of $f(x)$ *as* x *approaches* a *from the right equals* L*.*
Left-sided limit*: Suppose* f *is defined for all* x *near* a *with* $x < a$*. If* $f(x)$ *is arbitrarily close to* L *for all* x *sufficiently close to* a *with* $x < a$*, we write*

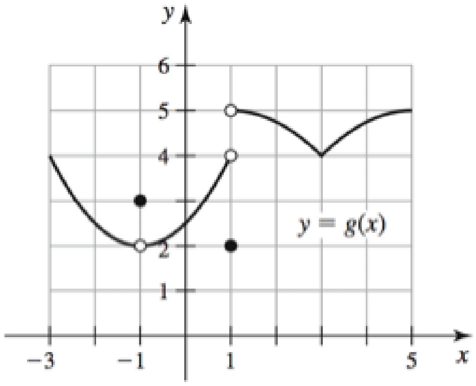
$$\lim_{x\to a^-} f(x) = L$$

and say that the limit of $f(x)$ *as* x *approaches* a *from the left equals* L*.*

Note (one-sided versus two-sided limits): The limit $\lim\limits_{x\to a} f(x) = L$ is a *two-sided limit* because $f(x)$ approaches L as x approaches a for values of x less than a *and* for values of x greater than a.

Theorem 5 (Relationship between one-sided and two-sided limits) *Assume* f *is defined for all* x *near* a *except possibly at* a*. Then* $\lim\limits_{x\to a} f(x) = L$ *if and only if* $\lim\limits_{x\to a^+} f(x) = L$ *and* $\lim_{x\to a^-} f(x) = L$*.*

Example 6 (§2.2 **Ex. 24**) *Use the graph of g in the figure to find the following values or state that they do not exist. If a limit does not exist, explain why.*



(a) $g(-1)$

(b) $\lim_{x\to -1^-} g(x)$

(c) $\lim_{x\to -1^+} g(x)$

(d) $\lim_{x\to -1} g(x)$

(e) $g(1)$

(f) $\lim_{x\to 1} g(x)$

(g) $\lim_{x\to 3} g(x)$

(h) $g(5)$

(i) $\lim_{x\to 5^-} g(x)$