Professor Jennifer Balakrishnan, *jbala@bu.edu*

What is on today

1 Derivatives of trigonometric functions

Briggs-Cochran-Gillett §3.5 pp. 163 – 171

1.1 Special trigonometric limits

In the same way that finding the derivative of e^x needed the special limit $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$, to find the derivatives of $\sin x$ and $\cos x$, we also need some limits that are not computed directly using the rules which we have already learned.

Theorem 1.
$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \quad \textit{and} \quad \lim_{x\to 0} \frac{\cos x - 1}{x} = 0.$$

Example 2 (§3.5 Ex. 13 and 16)**.** *Use the theorem above to evaluate the following limits.*

1. $\lim_{x\to 0} \frac{\tan 7x}{\sin x}$
2. $\lim_{x\to -3} \frac{\sin(x+3)}{x^2 + 8x + 15}$

1.2 Derivatives

Theorem 3 (Derivatives of trigonometric functions)**.**

1. $\dfrac{d}{dx}(\sin x) = \cos x$
2. $\dfrac{d}{dx}(\cos x) = -\sin x$
3. $\dfrac{d}{dx}(\tan x) = \sec^2 x$
4. $\dfrac{d}{dx}(\cot x) = -\csc^2 x$
5. $\dfrac{d}{dx}(\sec x) = \sec x \tan x$
6. $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$

Example 4 (§3.5 Ex. 18 and 24)**.** *Find dy/dx for the following functions.*

1. $y = 5x^2 + \cos x$
2. $y = \dfrac{1 - \sin x}{1 + \sin x}$

Example 5 (§3.5 based on Ex. 36)**.** *Find the derivative of* $y = \frac{\tan w}{1+\tan w}$

1. *...directly using the formulas above;*
2. *...using only the formulas for the derivatives of* $\sin x$ *and* $\cos x$*.*

Example 6 (§3.5 Ex. 66)**.**

a. *For what values of x does $g(x) = x - \sin x$ have a horizontal tangent line?*

b. *For what values of x does $g(x) = x - \sin x$ have a slope of 1?*

2 Derivatives as rates of change

Briggs-Cochran-Gillett §3.6 pp. 171–175

Definition 7 (Average and instantaneous velocity). *Let $s = f(t)$ be the position function of an object moving along a line. The average velocity of the object over the time interval $[a, a+\Delta t]$ is the slope of the secant line between $(a, f(a))$ and $(a+\Delta t, f(a+\Delta t))$:*

$$v_{av} = \frac{f(a+\Delta t) - f(a)}{\Delta t}.$$

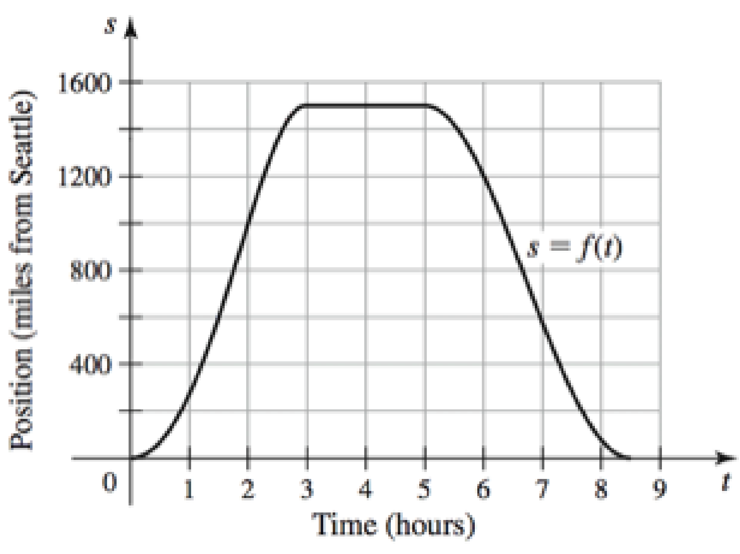
The instantaneous velocity at a is the slope of the line tangent to the position curve, which is the derivative of the position function:

$$v(a) = \lim_{\Delta t \to 0} \frac{f(a+\Delta t) - f(a)}{\Delta t} = f'(a).$$

Definition 8 (Velocity, speed, and acceleration). *Suppose an object moves along a line with position $s = f(t)$. Then*

- *the* ***velocity*** *at time t is $v = \frac{ds}{dt} = f'(t)$*
- *the* ***speed*** *at time t is $|v| = |f'(t)|$, and*
- *the* ***acceleration*** *at time t is $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = f''(t)$.*

Example 9 (§3.6 Ex. 10). *The following figure shows the position function of an airliner on an out-and-back trip from Seattle to Minneapolis, where $s = f(t)$ is the number of ground miles from Seattle t hours after take-off at 6:00 AM. The plane returns to Seattle 8.5 hours later at 2:30 PM.*



1. *Calculate the average velocity of the airliner during the first 1.5 hours of the trip* ($0 \le t \le 1.5$)*.*
2. *Calculate the average velocity of the airliner between 1:30 PM and 2:30 PM* ($7.5 \le t \le 8.5$)*.*
3. *At what time(s) is the velocity 0? Give a plausible explanation.*
4. *Determine the velocity of the airliner at noon* ($t = 6$) *and explain why the velocity is negative.*

Example 10 (§3.6 Ex. 14)**.** *Suppose the position of an object moving horizontally after t seconds is given by the function* $s = f(t)$*, where* $f(t) = 18t - 3t^2$ *with* s *measured in feet, with* $s > 0$ *corresponding to positions right of the origin. Consider the function on the interval* $0 \le t \le 8$.

1. *Graph the position function.*
2. *Find and graph the velocity function. When is the object stationary, moving to the right, and moving to the left?*
3. *Determine the velocity and acceleration of the object at* $t = 1$*.*
4. *Determine the acceleration of the object when its velocity is zero.*
5. *On what intervals is the speed increasing?*

Example 11 (§3.6 Ex. 18). *Suppose a stone is thrown vertically upward from the edge of a cliff on Mars (where the acceleration due to gravity is only about 12 ft/s^2) with an initial velocity of 64 ft/s from height of 192 ft above the ground. The height s of the stone above the ground after t seconds is given by $s = -6t^2 + 64t + 192$.*

1. *Determine the velocity v of the stone after t seconds.*
2. *When does the stone reach its highest point?*
3. *What is the height of the stone at the highest point?*
4. *When does the stone strike the ground?*
5. *With what velocity does the stone strike the ground?*