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What is on today

1 Chain Rule

Briggs-Cochran-Gillett §3.7 pp. 185–194

The *Chain Rule* lets us differentiate composite functions:

Theorem 1 (Chain Rule). *Suppose $y = f(u)$ is differentiable at $u = g(x)$ and $u = g(x)$ is differentiable at x. The composite function $y = f(g(x))$ is differentiable at x, and its derivative can be expressed in two equivalent ways:*
Version 1:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Version 2:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

Example 2 (§3.7 Ex. 8). *Use the Chain Rule to compute the derivative of $y = (5x^2 + 11x)^{20}$.*

Example 3 (§3.7 based on Ex. 10). *Use the Chain Rule to compute the derivatives of the following functions:*

1. $y = \cos x^5$
2. $y = \cos^5 x$

Example 4 (§3.7 Ex. 13, 28). *Calculate the derivatives of the following functions:*

1. $y = \sqrt{x^2 + 1}$
2. $y = e^{\tan t}$

Example 5 (§3.7 Ex. 38). *Let $h(x) = f(g(x))$ and $k(x) = g(g(x))$. Use the table to compute the following derivatives.*

1. $h'(1)$
2. $h'(2)$
3. $h'(3)$
4. $k'(3)$
5. $k'(1)$
6. $k'(5)$

x	1	2	3	4	5
$f'(x)$	−6	−3	8	7	2
$g(x)$	4	1	5	2	3
$g'(x)$	9	7	3	−1	−5

Example 6 (§3.7 Ex. 48, 52, 54, 64, 68). *Calculate the derivatives of the following functions:*

1. $y = \sin^2(e^{3x+1})$
2. $y = (1 - e^{-0.05x})^{-1}$
3. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$
4. $y = e^{2x}(2x - 7)^5$
5. $y = (z + 4)^3 \tan z$

Example 7 (§3.7 Ex. 82). *Suppose f is differentiable on $[-2, 2]$ with $f'(0) = 3$ and $f'(1) = 5$. Let $g(x) = f(\sin x)$. Evaluate the following expressions:*

1. $g'(0)$
2. $g'\left(\frac{\pi}{2}\right)$
3. $g'(\pi)$