Professor Jennifer Balakrishnan, *jbala@bu.edu*

What is on today

1 Derivatives of logarithmic and exponential functions

Briggs-Cochran-Gillett §3.9 pp. 203 – 213

Example 1 (§3.9 Ex. 52)**.** *Determine whether the graph of* $y = x^{\sqrt{x}}$ *has any horizontal tangent lines.*

Example 2 (§3.9 Ex. 72)**.** *Compute the following higher order derivatives:* $\frac{d^n}{dx^n}(2^x)$*.*

Example 3 (§3.9 Ex. 78)**.** *Let* $f(x) = \ln \frac{2x}{(x^2+1)^3}$*. Use the properties of logarithms to simplify the function before computing* $f'(x)$*.*

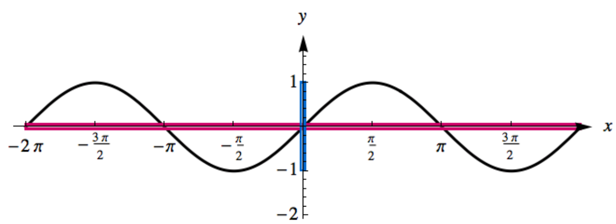
Example 4 (§3.9 Ex. 88)**.** *Compute the derivative* $\frac{d}{dx}(x^\pi + \pi^x)$*.*

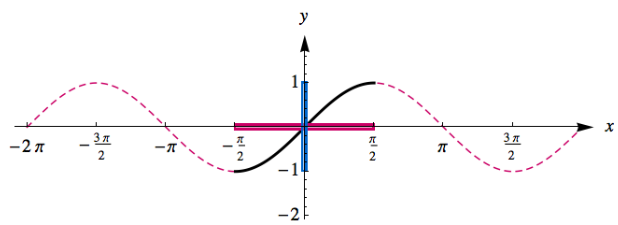
2 Review of Inverse Trigonometric Functions

Briggs-Cochran-Gillett §1.4 pp. 38 – 51

2.1 Sine and Arcsine

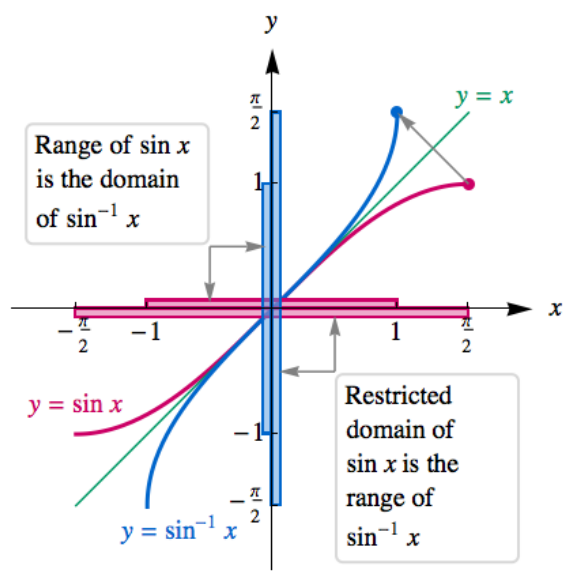
To invert a function f on a domain we need it to be one-to-one on that domain. This means that every output of the function f must correspond to exactly one input. (Recall that the one-to-one property is checked graphically by using the *horizontal line test*.) The function $\sin x$ is not one-to-one over all its domain, but if we restrict it to $[-\pi/2, \pi/2]$ it is one-to-one, and it makes sense to talk about its inverse.





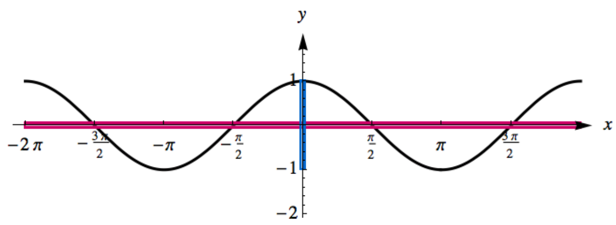
The inverse of $\sin x$ is $\arcsin x = \sin^{-1} x$.

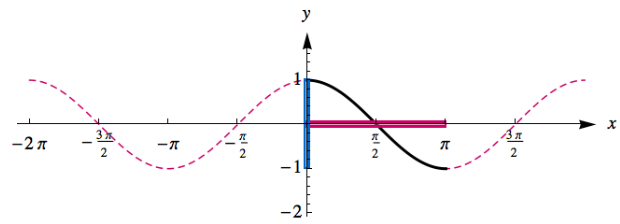
- $\sin^{-1}(x)$ is the angle whose sin is x
- Domain($\sin^{-1} x$)=$[-1, 1]$ (range of $\sin x$)
- Range($\sin^{-1} x$)=$[-\pi/2, \pi/2]$ (restricted domain of $\sin x$)
- Graphically the two functions are symmetric about the line $y = x$
- $\sin(\sin^{-1}(x)) = x$ for all x in $[-1, 1]$
- $\sin^{-1}(\sin(x)) = x$ for all x in $[-\frac{\pi}{2}, \frac{\pi}{2}]$
- Remark: $\sin^{-1} x$ **is not** $\dfrac{1}{\sin x}$



2.2 Cosine and Arccosine

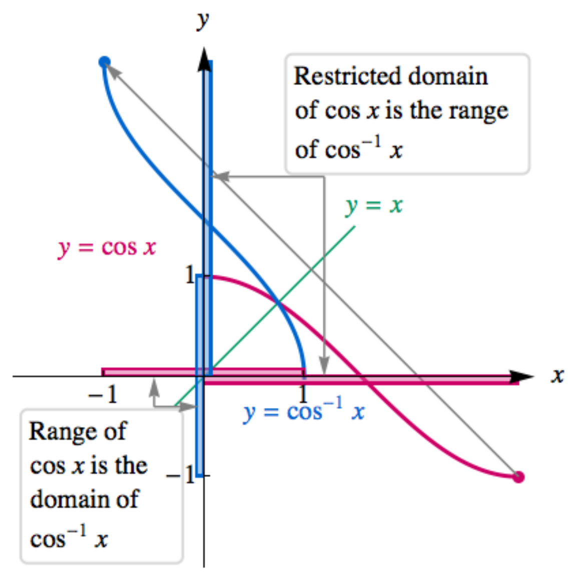
In the same way as above, the function $\cos x$ is not one-to-one over all its domain, but if we restrict it to $[0, \pi]$ it is one-to-one, and it makes sense to talk about its inverse.





The inverse of $\cos x$ is $\arccos x = \cos^{-1} x$.

- $\cos^{-1}(x)$ is the angle whose cos is x
- Domain $(\cos^{-1} x)$=$[-1, 1]$ (range of $\cos x$)
- Range $(\cos^{-1} x)$=$[0, \pi]$ (restricted domain of $\cos x$)
- Graphically the two functions are symmetric about the line $y = x$
- $\cos(\cos^{-1}(x)) = x$ for all x in $[-1, 1]$
- $\cos^{-1}(\cos(x)) = x$ for all x in $[0, \pi]$
- Remark: $\cos^{-1} x$ **is not** $\dfrac{1}{\cos x}$



2.3 Other inverse trig functions

We proceed in the same way to find the inverse functions to all trigonometric functions.

DEFINITION **Other Inverse Trigonometric Functions**

$y = \tan^{-1} x$ is the value of y such that $x = \tan y$, where $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$.

$y = \cot^{-1} x$ is the value of y such that $x = \cot y$, where $0 < y < \pi$.

The domain of both $\tan^{-1} x$ and $\cot^{-1} x$ is $\{x: -\infty < x < \infty\}$.

$y = \sec^{-1} x$ is the value of y such that $x = \sec y$, where $0 \le y \le \pi$, with $y \ne \dfrac{\pi}{2}$.

$y = \csc^{-1} x$ is the value of y such that $x = \csc y$, where $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$, with $y \ne 0$.

The domain of both $\sec^{-1} x$ and $\csc^{-1} x$ is $\{x: |x| \ge 1\}$.

2.4 Examples

Example 5 (§1.4 Ex. 47, 52, 53, 67, 68, 69)**.** *Evaluate the following expressions (without calculator!) or state that they are not defined.*

(i) $\sin^{-1}(1)$ *(iii)* $\cos^{-1}(-1/2)$ *(v)* $\cot^{-1}(-1/\sqrt{3})$

(ii) $\cos^{-1}(2)$ *(iv)* $\tan^{-1}(\sqrt{3})$ *(vi)* $\sec^{-1}(2)$

Example 6 (§1.4 Ex. 57, 59, 75)**.** *Simplify the given expressions. Assume $x > 0$.*

(i) $\cos(\sin^{-1} x)$ *(ii)* $\sin(\cos^{-1}(x/2))$ *(iii)* $cos(\tan^{-1} x)$

3 Derivatives of Inverse Trigonometric Functions

Briggs-Cochran-Gillett §3.10 pp. 214 – 223

Using implicit differentiation and trigonometric identities we get:

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}, \text{ for } -1 < x < 1$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2} \qquad \frac{d}{dx}\left(\cot^{-1} x\right) = -\frac{1}{1+x^2}, \text{ for } -\infty < x < \infty$$

$$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\left(\csc^{-1} x\right) = -\frac{1}{|x|\sqrt{x^2-1}}, \text{ for } |x| > 1$$

Example 7 (§3.10 Ex. 8, 16, 18, 20, 29). *Evaluate the derivatives of the following functions.*

1. $f(x) = x \sin^{-1} x$
2. $g(z) = \tan^{-1}(1/z)$
3. $f(x) = \sec^{-1} \sqrt{x}$
4. $f(t) = (\cos^{-1} t)^2$
5. $f(s) = \cot^{-1}(e^s)$