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What is on today

1 Derivatives of Inverse Trig Functions Wrap-up

Briggs-Cochran-Gillett §3.10 pp. 214 – 223

We saw during the last lecture:

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}, \text{ for } -1 < x < 1$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2} \qquad \frac{d}{dx}\left(\cot^{-1} x\right) = -\frac{1}{1+x^2}, \text{ for } -\infty < x < \infty$$

$$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\left(\csc^{-1} x\right) = -\frac{1}{|x|\sqrt{x^2-1}}, \text{ for } |x| > 1$$

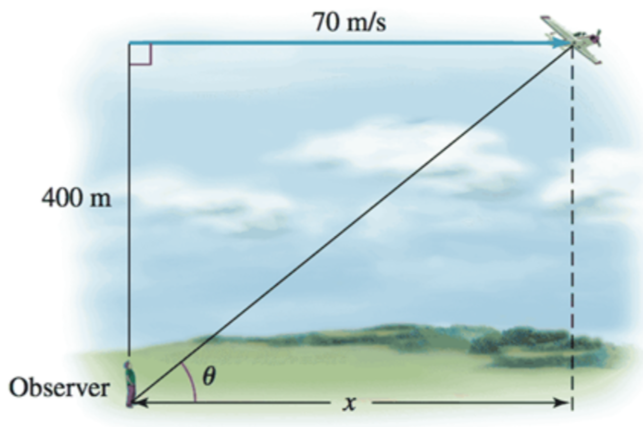
Example 1 (§3.10 Ex. 24, 28)**.** *Evaluate the derivatives of the following functions.*

1. $f(w) = \sin(\sec^{-1}(2w))$
2. $f(x) = \sin(\tan^{-1}(\ln x))$

Example 2 (§3.10 Ex. 34)**.** *Find an equation of the line tangent to the graph of* $f(x) = \sec^{-1}(e^x)$ *at the point* $\left(\ln 2, \frac{\pi}{3}\right)$.

1.1 Application

Example 3 (§3.10 Ex. 36). *A small plane, moving at 70 m/s, flies horizontally on a line 400 m directly above an observer. Let θ be the angle of elevation of the plane (see figure).*



a. *What is the rate of change of the angle of elevation $\frac{d\theta}{dx}$ when the plane is $x = 500$ m past the observer?*

b. *Graph $\frac{d\theta}{dx}$ as a function of x and determine the point at which θ changes most rapidly.*

2 Related rates

Briggs-Cochran-Gillett §3.11 pp. 224-231

We revisit the idea of the derivative as the rate of change of a function, in the context of problems that have related variables changing with respect to time.

Example 4 (§3.11 Ex. 6). *The sides of a square decrease in length at a rate of 1 m/s.*

1. *At what rate is the area of the square changing when the sides are 5 m long?*
2. *At what rate are the lengths of the diagonals of the square changing?*

Example 5 (§3.11 Ex. 20)**.** *A jet ascends at a* $10°$ *angle from the horizontal with an airspeed of 550 mi/hr (its speed along its line of flight is 550 mi/hr). How fast is the altitude of the jet increasing? If the sun is directly overhead, how fast is the shadow of the jet moving on the ground?*

Example 6 (§3.11 Ex. 24)**.** *A 12-foot ladder is leaning against a vertical wall when Jack begins pulling the foot of the ladder away from the wall at a rate of 0.2 ft/s. What is the configuration of the ladder at the instant that the vertical speed of the top of the ladder equals the horizontal speed of the foot of the ladder?*

Example 7 (§3.11 Ex. 26)**.** *Runners stand at first and second base in a baseball game. At the moment a ball is hit, the runner at first base runs to second base at 18 ft/s; simultaneously the runner on second runs to third base at 20 ft/s. How fast is the distance between the runners changing 1 second after the ball is hit (see figure)? (Hint: the distance between consecutive bases is 90 ft and the bases lie at the corners of a square.)*