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# What is on today



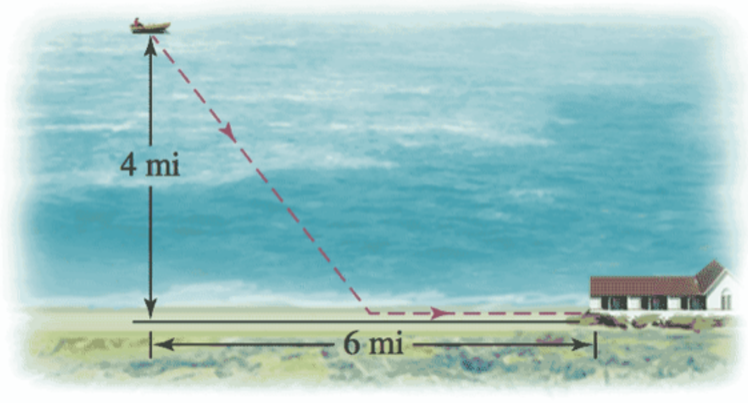
# 1 Optimization Problems

Briggs-Cochran-Gillett §4.4 pp. 270-280

We continue our exploration of problems in optimization.

**Example 1** (§4.4 Ex. 17: Shipping crates)**.** *A square-based, box-shaped shipping crate is designed to have a volume of* 16 $ft^3$*. The material used to make the base costs twice as much (per square foot) as the material in the sides, and the material to do the top costs half as much (per square foot) as the material in the sides. What are the dimensions of the crate that minimize the cost of materials?*

**Example 2** (§4.4 Ex. 21: Walking and rowing)**.** *A boat on the ocean is* 4 *mi from the nearest point on a straight shoreline; that point is* 6 *mi from a restaurant on the shore. A woman plans to row the boat straight to some point on the shore and then walk to the restaurant.*




1. *If she walks at* 3 *mi/hr and rows at* 2 *mi/hr, at which point on the shore should she land to minimize the total travel time?*
2. *If she walks at* 3 *mi/hr, what is the minimum speed at which she must row so that the quickest way to the restaurant is to row directly (with no walking)?*

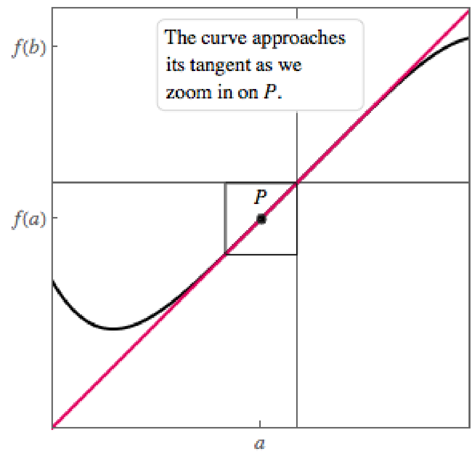
**Example 3** (§4.4 Ex. 54: Maximizing profit)**.** *Suppose you own a tour bus and you book groups of 20 to 70 people for a day tour. The cost per person is* \$30 *minus* \$0.25 *for every ticket sold. If gas and other miscellaneous costs are* \$200*, how many tickets should you sell to maximize your profit? Treat the number of tickets as a nonnegative real number.*

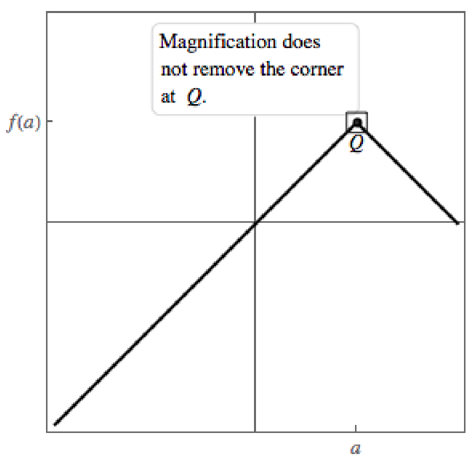
# 2 Linear approximation

Briggs-Cochran-Gillett §4.5 pp. 281 - 287

## 2.1 Tangent lines and the main idea

For a function that is differentiable at a point $P$, zooming in to the graph near $P$ yields a piece of the curve that looks more and more like the tangent line at $P$. This fundamental observation, that smooth curves appear straighter on smaller scales, is called local linearity. It is the basis of many important mathematical ideas, one of which is *linear approximation*.




On the other hand, if we consider a curve with a corner or cusp at a point $Q$, no amount of magnification "straightens out" the curve at $Q$. The different behavior at $P$ and $Q$ is related to the idea of differentiability. One of the requirements for the techniques in this section is that the function be differentiable at the point in question.

The idea of linear approximation is to use the line tangent to the curve at $P$ to approximate the value of the function at points near $P$.

**Definition 4** (Linear approximation to $f$ at $a$). *Suppose $f$ is differentiable on an interval $I$ containing the point $a$. The linear approximation to $f$ at $a$ is the linear function*

$$L(x) = f(a) + f'(a)(x - a),$$

*for $x$ in $I$.*

**Example 5** (§4.5 Ex. 14)**.** *Let* $f(x) = \sin x$ *and* $a = \pi/4$*.*

1. *Write the equation of the line that represents the linear approximation to the function at the given point* $a$*.*
2. *Graph the function and the linear approximation at* $a$*.*
3. *Use the linear approximation to estimate the value* $f(0.75)$*.*
4. *Compute the percent error in your approximation, 100|(approximation−exact)|/|exact|, where the exact value is given by a calculator.*

**Example 6** (§4.5 Ex. 21, 24)**.** *Use linear approximations to estimate the following quantities. Choose a value of* $a$ *that produces a small error.*

1. $1/203$
2. $\sqrt[3]{65}$