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What is on today

1 Working with integrals

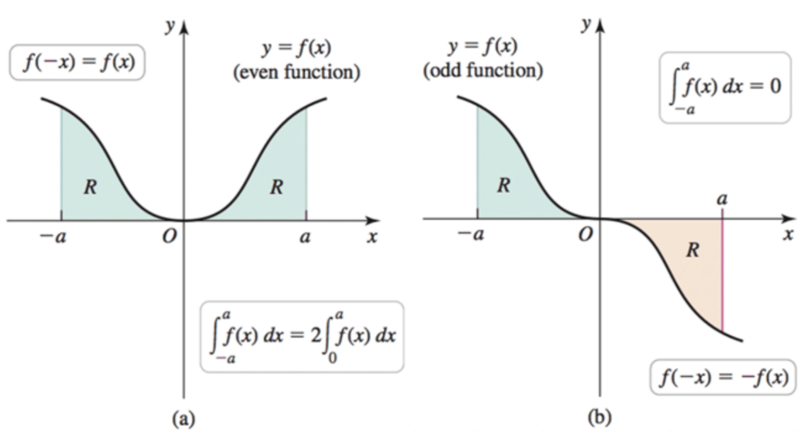
Briggs-Cochran-Gillett §5.4 (without Fundamental Theorem of Calculus) pp. 377-384

1.1 Even and odd functions

Definition 1 (Even and odd functions)**.**

1. *An* ***even*** *function f is a function that satisfies $f(-x) = f(x)$. This means its graph is symmetric about the y-axis. Ex:* $\cos(x)$, x^2, x^4.

2. *An* ***odd*** *function f is a function that satisfies $f(-x) = -f(x)$. This means its graph is symmetric about the origin. Ex:* $\sin(x)$, x, x^3.



Theorem 2. *Let a be a positive number and let f be an integrable function on the interval $[-a, a]$. Then*

1. *if f is even,* $\displaystyle\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$;

2. *if f is odd,* $\displaystyle\int_{-a}^{a} f(x)dx = 0.$

Example 3 (§5.4 Ex. 16)**.** *Use symmetry to evaluate the following integral*

$$\int_{-1}^{1} (1 - |x|)dx.$$

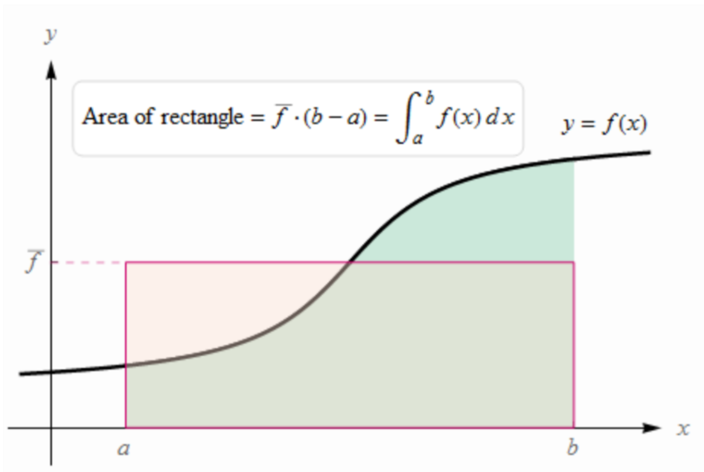
Example 4 (§5.4 Ex. 52)**.** *Suppose f is an odd function,* $\int_0^4 f(x)dx = 3$, *and* $\int_0^8 f(x)dx = 9$. *Evaluate*

1. $\int_{-4}^{8} f(x)dx$
2. $\int_{-8}^{4} f(x)dx$

1.2 Average value of a function

Definition 5 (Average value of a function)**.** *The average value of an integrable function on the interval $[a,b]$ is*

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)dx.$$

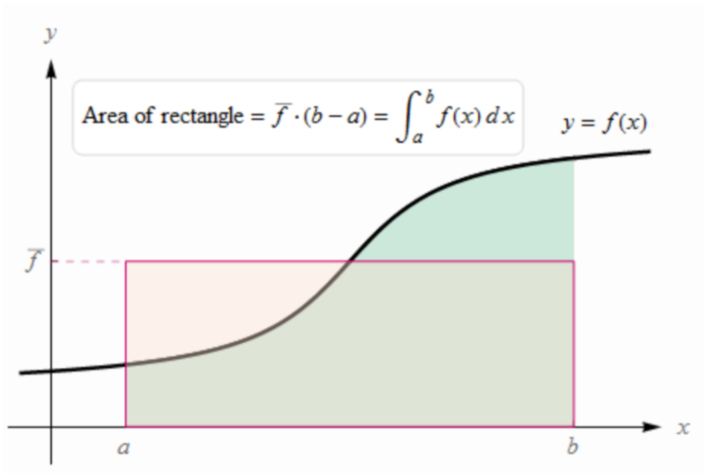


Example 6 (§5.4 Ex. 27)**.** *Find the average value of $f(x) = \cos x$ on $[-\pi/2, \pi/2]$ knowing that $\int_0^{\pi/2} \cos x \, dx = 1$. Draw a graph of the function and indicate the average value.*

1.3 Mean Value Theorem for integrals

Theorem 7. *Let f be continuous on the interval $[a,b]$. There exists a point c in (a,b) such that*

$$f(c) = \bar{f} = \frac{1}{b-a}\int_a^b f(x)dx.$$



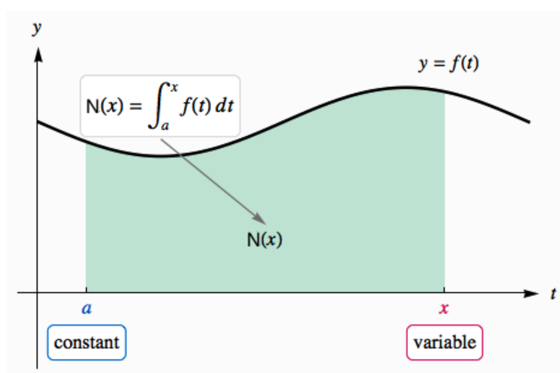
Example 8 (§5.4 Ex. 39). *Find or approximate all points at which the function $f(x) = 1-|x|$ equals its average value on the interval $[-1,1]$.*

2 Net area functions and the Fundamental Theorem of Calculus

Briggs-Cochran-Gillett §5.3 pp. 362-376

2.1 Net area functions

The concept of net area is essential in understanding the relationship between derivatives and integrals. If instead of finding the net area of a continuous function over a fixed interval $[a,b]$, we allow the right boundary point to move and calculate the net area over the intervals $[a,x]$, the net area for the different values of f define a function:



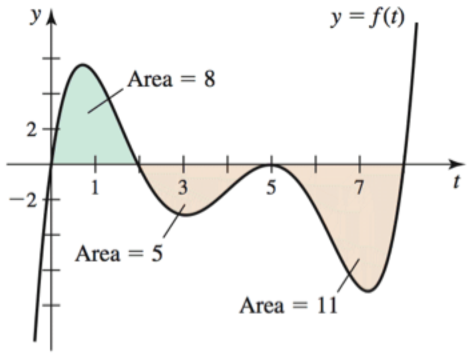
Definition 9. *Let f be a continuous function, for $t \geq a$. The* ***net area function for* f *with left endpoint* a** *is*

$$N(x) = \int_a^x f(t)dt,$$

where $x \geq a$.

Remark 10. *In the textbook, "net area" functions are called "area functions." We call them* ***net area*** *functions to make it clear that they calculate a* net *area and not an area!*

Example 11 (§5.3 Ex. 12).



The graph of f is shown in the figure. Let $N(x) = \int_0^x f(t)dt$ and $F(x) = \int_2^x f(t)dt$ be two net area functions for f. Evaluate

1. $N(2)$ *3.* $N(0)$ *5.* $N(8)$ *7.* $F(2)$

2. $F(5)$ *4.* $F(8)$ *6.* $N(5)$

Next time, we will discuss the Fundamental Theorem of Calculus and how it relates to net area functions.