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What is on today

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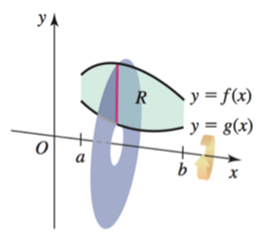
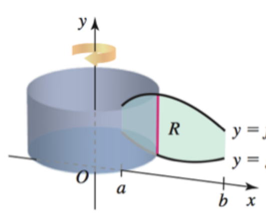
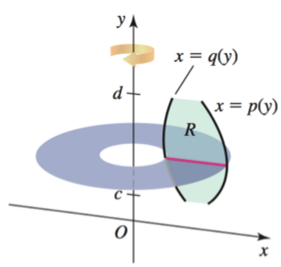
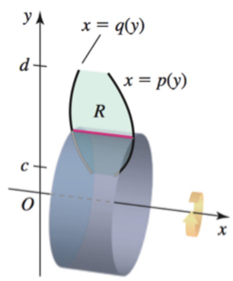
1 Volumes, wrap up

In the last class, we discussed several techniques (§§6.3-6.4) for computing volumes (slices, disks, washers, shells).

Integration with respect to *x* 	**Disk/washer method about the *x*–axis** Disks/washers are *perpendicular* to the *x*–axis. $\int_a^b \pi\left(f(x)^2 - g(x)^2\right) dx$
	**Shell method about the *y*–axis** Shells are *parallel* to the *y*–axis. $\int_a^b 2\pi x\,(f(x) - g(x))\, dx$
Integration with respect to *y* 	**Disk/washer method about the *y*–axis** Disks/washers are *perpendicular* to the *y*–axis. $\int_c^d \pi\left(p(y)^2 - q(y)^2\right) dy$
	**Shell method about the *x*–axis** Shells are *parallel* to the *x*–axis. $\int_c^d 2\pi y\,(p(y) - q(y))\, dy$

How do you choose a method, and which method is best? Note that the disk method is a special case of the washer method. That is, for solids of revolution, the choice is between the washer method and the shell method. In principle, either method can be used for any given problem, but in practice, one method usually produces an integral that is easier to evaluate than the other.

Example 1 (§6.4 Ex. 41)**.** *Let R be the region bounded by the curves $y = x$ and $y = x^{1/3}$ in the first quadrant, revolved about the x-axis. Let S be the solid generated. If possible, find the volume of S by both the washer and shell methods.*

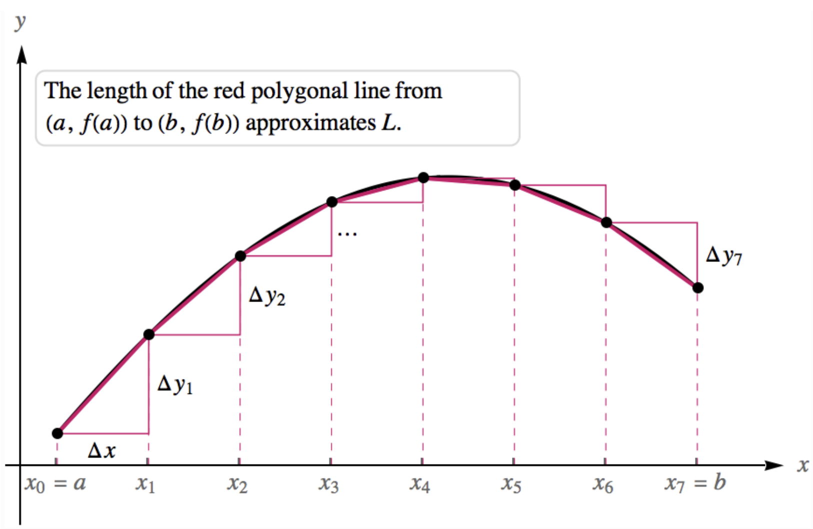
Example 2 (§6.4 Ex 46)**.** *Let R be the region bounded by the curves $y = 6/(x + 3)$ and $y = 2 - x$, revolved about the x-axis. Let S be the solid generated. If possible, find the volume of S by both the washer and shell methods.*

Example 3 (§6.4 Ex 48). *Let R be the region bounded by the curve $y = x - x^4$ and $y = 0$, revolved about the y-axis. Let S be the solid generated. If possible, find the volume of S by both the washer and shell methods.*

2 Lengths of curves

Briggs-Cochran-Gillett §6.5 pp. 445-449

Suppose a curve is given by $y = f(x)$, where f is a function with a continuous first derivative on the interval $[a, b]$. How far would you travel if you walked along the curve from $(a, f(a))$ to $(b, f(b))$? This distance is the arc length L.



By computing the lengths of the hypotenuses of the right triangles and taking the limit as the number of triangles goes to infinity, we obtain the following:

Definition 4. *Let f have a continuous first derivative on $[a, b]$. The length of the curve from $(a, f(a))$ to $(b, f(b))$ is*

$$L = \int_a^b \sqrt{1 + (f'(x))^2}dx.$$

Sometimes it is advantageous to describe a curve as a function of y: that is, $x = g(y)$. The arc length formula in this case is derived exactly as in the case of $y = f(x)$, switching the roles of x and y. This gives the following:

Definition 5. *Let $x = g(y)$ have a continuous first derivative on the interval $[c, d]$. The length of the curve from $(g(c), c)$ to $(g(d), d)$ is*

$$L = \int_c^d \sqrt{1 + (g'(y))^2}\, dy.$$

Example 6 (§6.5 Ex. 12)**.** *Find the arc length of the curve $y = 3\ln x - \frac{x^2}{24}$ on the interval $[1, 6]$.*

Example 7 (§6.5 Ex. 16)**.** *Find the arc length of the curve $y = \frac{2}{3}x^{3/2} - \frac{1}{2}x^{1/2}$ on $[1, 9]$.*

Example 8 (§6.5 Ex. 28)**.** *Find the arc length of the curve $y = \ln(x - \sqrt{x^2 - 1})$ for $1 \le x \le \sqrt{2}$ by integrating with respect to y.*