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What is on today

1 Exponential models

Briggs-Cochran-Gillett §6.9 pp. 482 - 488

Exponential functions are used to model problems in a number of fields: finance, medicine, ecology, biology, physics, to name a few. In this section, we study exponential models.

Exponential growth functions have the form

$$y(t) = y_0 e^{kt},$$

where y_0 is a constant and the rate constant k is positive. If we start with such a function and take its derivative, we find

$$y'(t) = ky_0 e^{kt} = ky.$$

We see that the growth rate $y'(t)$ is proportional to the value of the function. Another interesting quantity to consider is the relative growth rate $y'(t)/y(t)$, which is constant for exponential functions. Note that the initial value $y(0) = y_0$ and the rate constant determine the exponential function completely.

The quantity described by the function $y(t) = y_0 e^{kt}$ for $k > 0$ has a constant doubling time of $T_2 = \frac{\ln 2}{k}$.

Exponential decay is described by functions of the form $y(t) = y_0 e^{-kt}$. The initial value of y is $y(0) = y_0$ and the rate constant $k > 0$ determines the rate of decay. Exponential decay is characterized by a constant relative decay rate. The constant half-life is $T_{1/2} = \frac{\ln 2}{k}$.

Example 1 (§6.9 Ex. 29)**.** *Uranium-238 (U-238) has a half-life of 4.5 billion years. Geologists find a rock containing a mixture of U-238 and lead, and determine that 85% of the original U-238 remains; the other 15% has decayed into lead. How old is the rock?*

2 Basic approaches to integration

Briggs-Cochran-Gillett §7.1 pp. 511 - 514

In this section, we review some integration techniques. Here is a table of frequently used derivatives and antiderivatives:

$\frac{d}{du} F(u) = f(u)$	$\int f(u)\,du = F(u) + C$
$\frac{d}{du} u^{n+1} = (n+1)u^n$	$\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{du} \ln u = \frac{1}{u}$	$\int \frac{1}{u}\,du = \ln\lvert u\rvert + C$
$\frac{d}{du} \sin u = \cos u$	$\int \cos u\,du = \sin u + C$
$\frac{d}{du} \cos u = -\sin u$	$\int \sin u\,du = -\cos u + C$
$\frac{d}{du} \tan u = \sec^2 u$	$\int \sec^2 u\,du = \tan u + C$
$\frac{d}{du} \sec u = \sec u\, \tan u$	$\int \sec u\, \tan u\,du = \sec u + C$
$\frac{d}{du} e^u = e^u$	$\int e^u\,du = e^u + C$
	$\int \tan u\,du = \ln\lvert \sec u\rvert + C$
	$\int \sec u\,du = \ln\lvert \sec u + \tan u\rvert + C$
$\frac{d}{du} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}}$	$\int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1} u + C$
$\frac{d}{du} \tan^{-1} u = \frac{1}{1+u^2}$	$\int \frac{1}{1+u^2}\,du = \tan^{-1} u + C$

Example 2 (§7.1 Ex. 27). *Compute* $\int \frac{2-3x}{\sqrt{1-x^2}} dx$.

Example 3 (§7.1 Ex. 30). *Evaluate* $\int_2^4 \frac{x^2+2}{x-1}dx$.

Example 4 (§7.1 Ex. 34). *Evaluate* $\int_0^2 \frac{x}{x^2+4x+8}dx$.

Example 5 (§7.1 Ex. 61). *Consider the region* R *bounded by the graph of* $f(x) = \sqrt{x^2+1}$ *and the* x*-axis on the interval* $[0, 2]$.

1. *Find the volume of the solid formed when* R *is revolved about the* x*-axis.*
2. *Find the volume of the solid formed when* R *is revolved about the* y*-axis.*

3 Integration by parts

Briggs-Cochran-Gillett §7.2 pp. 516 - 520

The technique of integration by parts comes from reversing the product rule for derivatives:

Suppose that u, v are differentiable functions. Then

$$\int udv = uv - \int vdu$$

$$\int_a^b u(x)v'(x)dx = u(x)v(x)|_a^b - \int_a^b v(x)u'(x)dx.$$

The key is to figure out which function should be u and which one dv.

Example 6. *Evaluate* $\int \ln x \, dx$.

Example 7 (§7.2 Ex. 9)**.** *Evaluate* $\int te^t \, dt$.

Example 8 (§7.2 Ex. 15)**.** *Evaluate* $\int x^2 \ln x \, dx$.

Example 9 (§7.2 Ex. 29). *Evaluate* $\int x^2 \sin(2x)\,dx$.

Example 10 (§7.2 Ex. 34). *Evaluate* $\int_0^{\ln 2} xe^x\,dx$.

Example 11 (§7.2 Ex. 41). *Find the volume of the solid that is generated when the region bounded by* $f(x) = x \ln x$ *and the* x*-axis on* $[1, \mathrm{e}^2]$ *is revolved about the* x*-axis.*