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What is on today

1 Trigonometric integrals

Briggs-Cochran-Gillett §7.3 pp. 523 - 529

Evaluating $\int \sin^m x \cos^n x \, dx$

- If m is odd and positive and n is real: split off $\sin x$, rewrite the resulting even power of $\sin x$ in terms of $\cos x$, and then use $u = \cos x$.
- If n is odd and positive and m is real: split off $\cos x$, rewrite the resulting even power of $\cos x$ in terms of $\sin x$ and then use $u = \sin x$.
- If m and n are both even, nonnegative integers: use half-angle formulas to transform the integrand into a polynomial in $\cos 2x$ and apply the preceding strategies once again to powers of $\cos 2x$ greater than 1.

Half-angle formulas

$$\sin^2 x = \frac{1-\cos(2x)}{2} \qquad \cos^2 x = \frac{1+\cos(2x)}{2}$$

Example 1 (§7.3 Ex 12). *Evaluate* $\int \cos^4(2\theta)\, d\theta$.

Example 2 (§7.3 Ex 18). *Evaluate* $\int \sin^2\theta\cos^5\theta\,d\theta$.

Reduction formulas
Let n be a positive integer.

1. $\int \sin^n x\,dx = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
2. $\int \cos^n x\,dx = \frac{\cos^{n-1} x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
3. $\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx, n \neq 1$
4. $\int \sec^n x\,dx = \frac{\sec^{n-2} x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx, n \neq 1$

Integrals of $\tan x, \cot x, \sec x, \csc x$

$$\int \tan x\,dx = \ln|\sec x| + C \qquad \int \cot x\,dx = \ln|\sin x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C \qquad \int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

Evaluating $\int \tan^m x\sec^n x\,dx$

- If n is even: split off $\sec^2 x$, rewrite the remaining even power of $\sec x$ in terms of $\tan x$, and use $u = \tan x$.
- If m is odd: split off $\sec x\tan x$, rewrite the remaining even power of $\tan x$ in terms of $\sec x$, and use $u = \sec x$.
- If m is even and n is odd: rewrite the even power of $\tan x$ in terms of $\sec x$ to produce a polynomial in $\sec x$; apply reduction formula 4 to each term.

Example 3 (§7.3 Ex 42). *Evaluate* $\int \tan^5\theta \sec^4\theta \, d\theta$.

2 Trigonometric substitution

Briggs-Cochran-Gillett §7.4 pp. 531 - 533

For integrals with an $a^2 - x^2$ term, we make the trigonometric substitution $x = a\sin\theta$; note that this gives $\theta = \sin^{-1}(x/a)$ for $-\pi/2 \le \theta \le \pi/2$.

Example 4 (§7.4 Ex. 10). *Evaluate* $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx$.

Example 5 (§7.4 Ex 13). *Evaluate* $\int \frac{dx}{(16-x^2)^{1/2}}$.

Example 6 (§7.4 Ex 32). *Evaluate* $\int \sqrt{9-4x^2}\, dx$.