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# What is on today

1 More on sequences 1

# 1 More on sequences

Briggs-Cochran-Gillett §8.2 pp. 607 - 611

**Theorem 1** (Squeeze Theorem for sequences)**.** *Let $\{a_n\}, \{b_n\}$, and $\{c_n\}$ be sequences with $a_n \leq b_n \leq c_n$ for all integers $n$ greater than some index $N$. If $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then $\lim_{n\to\infty} b_n = L$.*

**Example 2** (§8.2 Ex 32)**.** *Find the limit of the sequence $\{\frac{(-1)^n}{n}\}$ or determine that the limit does not exist.*

We now introduce some terminology for sequences:

- $\{a_n\}$ is increasing if $a_{n+1} > a_n$; for example, $\{0, 1, 2, 3, \ldots\}$,
- $\{a_n\}$ is nondecreasing if $a_{n+1} \geq a_n$; for example, $\{1, 1, 2, 2, 3, 3, \ldots\}$.
- $\{a_n\}$ is decreasing if $a_{n+1} < a_n$; for example, $\{2, 1, 0, -1, \ldots\}$.
- $\{a_n\}$ is nonincreasing if $a_{n+1} \leq a_n$; for example, $\{0, -1, -1, -2, -2, -3, -3, \ldots\}$.
- $\{a_n\}$ is monotonic if it is either nonincreasing or nondecreasing (it moves in one direction).
- $\{a_n\}$ is bounded if there is a number $M$ such that $|a_n| \leq M$, for all relevant values of $n$

**Geometric sequences** have the property that each term is obtained by multiplying the previous term by a fixed constant, called the ratio. They have the form $\{ar^n\}$, where the ratio $r$ and $a \neq 0$ are real numbers.

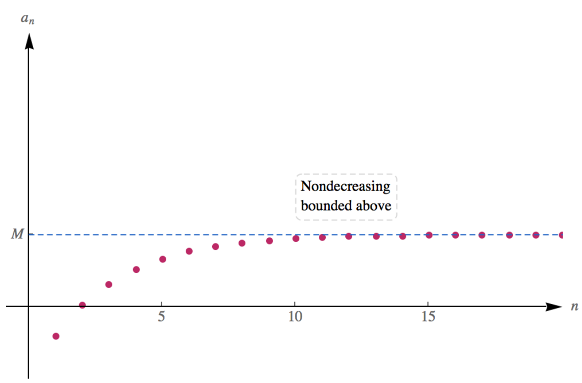
**Theorem 3.** *Let $r$ be a real number. Then*

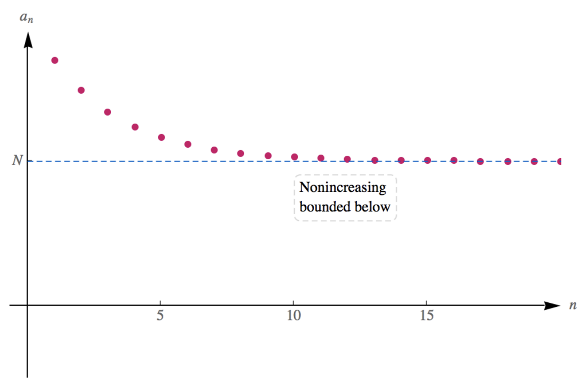
$$\lim_{n\to\infty} r^n = \begin{cases} 0 & \text{if } |r| < 1 \\ 1 & \text{if } r = 1 \\ \text{does not exist} & \text{if } r \le -1 \text{ or } r > 1. \end{cases}$$

*If $r > 0$, then $\{r^n\}$ is a monotonic sequence. If $r < 0$, then $\{r^n\}$ oscillates.*

**Example 4** (§8.2 Ex 50)**.** *Determine whether the sequence $\{2^{n+1}3^{-n}\}$ converges or diverges, and state whether it is monotonic or whether it oscillates. Give the limit if the sequence converges.*

Consider the following bounded monotonic sequences:




Indeed, it is a theorem that all such sequences converge:

**Theorem 5.** *A bounded monotonic sequence converges.*

We will review for Midterm 1 for the rest of the lecture.