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What is on today

1 Properties of power series

Briggs-Cochran-Gillett §9.2 pp. 675 - 679

We saw that Taylor polynomials provide accurate approximations to many functions, and that, in general, the approximations improve as the degree of the polynomials increase. Today we will let the degree of the polynomial increase without bound and produce a power series.

One way to start thinking about power series is to revisit the idea of geometric series: recall that if we fix a real number r such that $|r| < 1$, we have

$$\sum_{k=0}^{\infty} r^k = 1 + r + r^2 + \cdots = \frac{1}{1-r}.$$

Now replace the real number r by a variable x. Then we have the following statement:

$$\sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \cdots = \frac{1}{1-x}, \text{ if } |x| < 1.$$

This infinite series is an example of a power series, which we define more formally below.

Definition 1. *A power series has the general form*

$$\sum_{k=0}^{\infty} c_k (x-a)^k$$

where a and c_k are real numbers and x is a variable.

- *The c_k are coefficients of the power series and a is the center of the power series.*
- *The set of values of x for which the series converges is its interval of convergence.*
- *The radius of convergence of the power series, denoted R, is the distance from the center of the series to the boundary of the interval of convergence.*

We use the Ratio Test or Root Test to determine the interval of convergence for a given power series. We illustrate this in a few examples.

Example 2 (§9.2 Ex 10, 18, 24)**.** *Determine the radius of convergence of the following power series. Then test the endpoints to determine the interval of convergence.*

1. $\sum \frac{(2x)^k}{k!}$

2. $\sum (-1)^k \frac{x^k}{5^k}$

3. $\sum \left(-\frac{x}{10}\right)^{2k}$

Example 3 (§9.2 Ex 32, 34)**.** *Use the geometric series*

$$f(x) = \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

for $|x| < 1$ *to find the power series representation for the following functions centered at 0. Give the interval of convergence of the new series.*

1. $f(x^3) = \frac{1}{1-x^3}$

2. $f(-4x) = \frac{1}{1+4x}$

Example 4 (§9.2 Ex 36)**.** *Use the power series representation*

$$f(x) = \ln(1-x) = -\sum_{k=1}^{\infty} \frac{x^k}{k}$$

for $-1 \leq x < 1$ *to find the power series for the function* $g(x) = x^3 \ln(1-x)$ *centered at 0. Give the interval of convergence of the new series.*