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What is on today

1 Properties of power series

Briggs-Cochran-Gillett §9.2 pp. 675 - 679

Example 1 (§9.2 Ex 38). *Use the power series representation*

$$f(x) = \ln(1-x) = -\sum_{k=1}^{\infty} \frac{x^k}{k}$$

for $-1 \leq x < 1$ *to find the power series for the function* $f(x^3) = \ln(1-x^3)$ *centered at 0. Give the interval of convergence of the new series.*

Theorem 2 (Combining power series). *Suppose the power series* $\sum c_k x^k$ *and* $\sum d_k x^k$ *converge to* $f(x)$ *and* $g(x)$*, respectively, on an interval* I*.*

1. *Sum and difference: The power series* $\sum (c_k \pm d_k) x^k$ *converges to* $f(x) \pm g(x)$ *on* I*.*
2. *Multiplication by a power: Suppose* m *is an integer such that* $k + m \geq 0$ *for all terms of the power series* $x^m \sum c_k x^k = \sum c_k x^{k+m}$*. This series converges to* $x^m f(x)$ *for all* $x \neq 0$ *in* I*. When* $x = 0$*, the series converges to* $\lim_{x \to 0} x^m f(x)$*.*
3. *Composition: If* $h(x) = bx^m$*, where* m *is a positive integer and* b *is a nonzero real number, the power series* $\sum c_k (h(x))^k$ *converges to the composite function* $f(h(x))$*, for all* x *such that* $h(x)$ *is in* I*.*

Theorem 3 (Differentiating and integrating power series). *Suppose the power series* $\sum c_k (x-a)^k$ *converges for* $|x - a| < R$ *and defines a function* f *on that interval.*

1. *Then f is differentiable (which implies continuous) for $|x-a|<R$, and f' is found by differentiating the power series for f term by term; that is, $f'(x)=\sum kc_k(x-a)^{k-1}$, for $|x-a|<R$.*

2. *The indefinite integral of f is found by integrating the power series for f term by term; that is, $\int f(x)dx=\sum c_k\frac{(x-a)^{k+1}}{k+1}+C$, for $|x-a|<R$, where C is an arbitrary constant.*

Example 4 (§9.2 Ex 42, 44, 46)**.** *Find the power series representation for g centered at 0 by differentiating or integrating the power series for f (perhaps more than once). Give the interval of convergence for the resulting series.*

1. $g(x)=\frac{1}{(1-x)^3}$ *using* $f(x)=\frac{1}{1-x}$

2. $g(x)=\frac{x}{(1+x^2)^2}$ *using* $f(x)=\frac{1}{1+x^2}$

3. $g(x)=\ln(1+x^2)$ *using* $f(x)=\frac{x}{1+x^2}$

Example 5 (§9.2 Ex 48, 50)**.** *Find power series representations centered at 0 for the following functions using known power series. Give the interval of convergence for the resulting series.*

1. $f(x) = \frac{1}{1-x^4}$

2. $f(x) = \ln \sqrt{1 - x^2}$

Example 6 (§9.2 Ex 64)**.** *Find the function represented by the series* $\sum_{k=1}^{\infty} \frac{x^{2k}}{4k}$ *and find the interval of convergence of the series.*