Professor Jennifer Balakrishnan, *jbala@bu.edu*

What is on today

1 Working with Taylor series

Briggs-Cochran-Gillett §9.4 pp. 699 - 702

We wrap up our study of Taylor series today.

Example 1 (§9.4 Ex 41). *Use a Taylor series to approximate the definite integral*

$$\int_0^{.35} \tan^{-1} x \, dx.$$

Use as many terms as needed to ensure the error is less than 10^{-4}.

Example 2 (§9.4 Ex 55, 62). *Identify the functions represented by the following power series.*

1. $\sum_{k=0}^{\infty} \frac{x^k}{2^k}$

2. $\sum_{k=1}^{\infty} \frac{x^{2k}}{k}$

2 Differential equations

Briggs-Cochran-Gillett §7.9 pp. 581 - 585

Differential equations are a powerful tool used in engineering, the natural and biological sciences, economics, management and finance. As our last topic in this course, we give a brief introduction to differential equations.

A differential equation is an equation involving a function and its derivative: for example, $y'(t) = 3t^2 - 4t + 10$. The goal is to find solutions of the equation: functions y that satisfy the equation.

Now we introduce some terminology for working with differential equations. The **order** of a differential equation is the highest order appearing on a derivative in the equation. For example, $y'' + 16y = 0$ is second order.

A **first-order linear differential equation** has the form

$$y'(x) + p(x)y(x) = f(x)$$

where p, q, f are given functions that depend only on the independent variable x. A differential equation is often accompanied by initial conditions that specify the values of y, and possibly its derivatives, at a particular point. In general, an nth order equation requires n initial conditions. A differential equation together with the appropriate number of initial conditions is called an **initial value problem**.

Example 3 (§7.9 Ex 17, 18)**.** *Solve the following initial value problems.*

1. $y'(t) = 3t^2 - 4t + 10, y(0) = 20$

2. $\frac{dy}{dt} = 8e^{-4t} + 1, y(0) = 5$

For any differential equation, the largest family of solutions, generated by arbitrary constants, is called the **general solution**. Below we describe the general solution of a first-order linear differential equation.

Theorem 4 (Solution of a first-order linear differential equation)**.** *The general solution of the first-order linear equation $y'(t) = ky + b$, where k and b are specified real numbers is*

$$y = Ce^{kt} - b/k,$$

where C is a constant. Given an initial condition, the value of C may be determined.

Example 5 (§7.9 Ex 22)**.** *Find the general solution of $\frac{dy}{dx} = -y + 2$.*

Example 6 (§7.9 Ex 26)**.** *Solve the initial value problem $\frac{dy}{dx} = -y + 2, y(0) = -2$.*

A **separable** differential equation is one that can be written as

$$g(y)y'(t) = h(t),$$

where the terms that involve y appear on one side of the equation separated from the terms that involve x. In general, we solve the separable equation $g(y)y'(t) = h(t)$ by integrating both sides of the equation with respect to t:

$$\begin{aligned} \int g(y)y'(t)dt &= \int h(t)dt \\ \int g(y)dy &= \int h(t)dt. \end{aligned}$$

Thus by changing variables on the left side of the equation, the solution relies on evaluating two integrals.

Example 7 (§7.9 Ex 35, 39)**.** *Determine whether the following equations are separable. If so, solve the given initial value problem.*

1. $\frac{dy}{dt} = ty + 2, y(1) = 2$

2. $\frac{dy}{dx} = e^{x-y}, y(0) = \ln 3$