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What is on today

1 Differential equations wrap up

Briggs-Cochran-Gillett §7.9 pp. 586 - 589

We continue our study of differential equations by looking at logistic population growth. The logistic equation is used to describe the population of many different species as well as the spread of rumors and epidemics.

Example 1 (§7.9 Ex 42)**.** *When an infected person is introduced into a closed and otherwise healthy community, the number of people who become infected with the disease (in the absence of any intervention) may be modeled by the logistic equation*

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{A}\right), \quad P(0) = P_0,$$

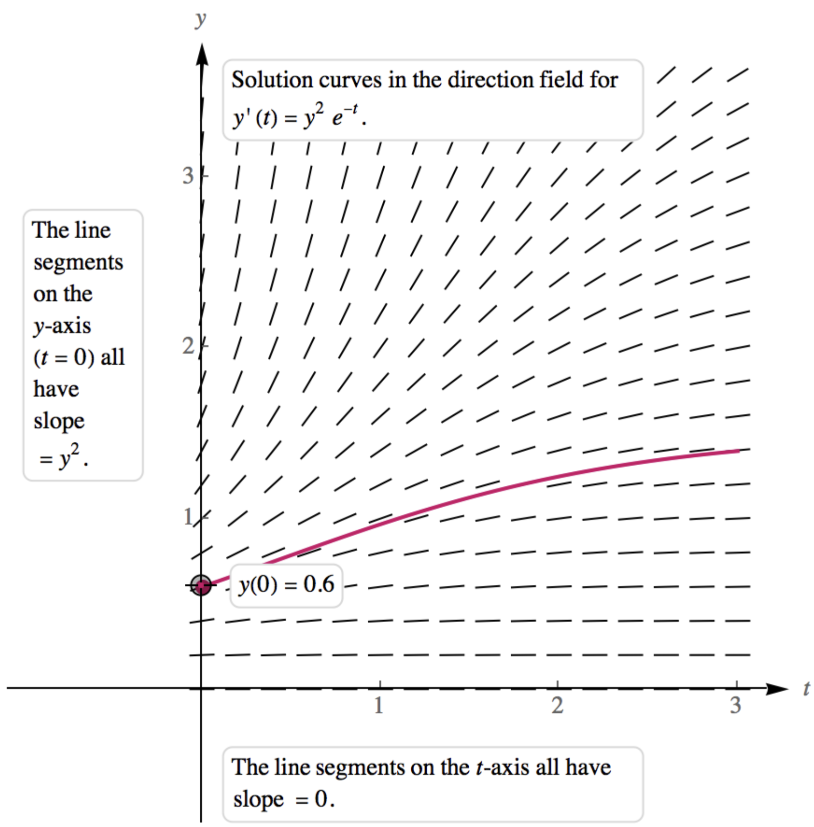
where K is a positive infection rate, A is the number of people in the community, and P_0 is the number of infected people at $t = 0$. The model assumes no recovery or intervention.

1. *Find the solution of the initial value problem in terms of $k, A,$ and P_0.*

2. *For fixed values of k and A, describe the long term behavior of the solutions for any P_0 with $0 < P_0 < A$.*

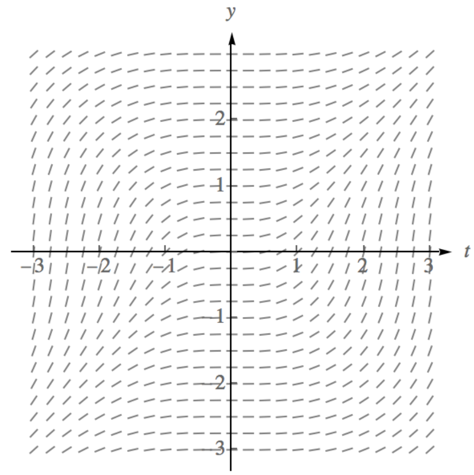
We can understand first-order differential equations geometrically using direction fields. If we consider the first-order differential equation $y'(t) = F(t, y)$, where F is a given expression involving t and or y, a solution of this equation has the property that at each point (t, y) of the solution curve, the slope of the curve is $F(t, y)$. A direction field is a picture that shows the slope of the solution at selected points of the ty-plane.

For example, let's look at the differential equation $y'(t) = y^2 e^{-t}$. At each point (t, y), we make a small line segment with slope $y^2 e^{-t}$. The line segment at a point P gives the slope of the solution curve that passes through P, as seen in the figure below. For example, along the t-axis ($y = 0$) the slopes of the line segments are $F(t, 0) = 0$. And along the y-axis ($t = 0$), the slopes of the line segments are $F(0, y) = y^2$.



Now suppose an initial condition $y(a) = A$ is given. Above, you can see the curve given by the initial condition of $y(0) = 0.6$. This picks out one particular solution curve. A different initial condition gives a different solution curve.

Example 2 (§7.9 Ex 43)**.** *Consider* $y'(t) = \frac{t^2}{y^2+1}$ *with the initial condition of* $y(0) = -2$. *Sketch a graph of the solution given the direction field below. Repeat this for the initial condition of* $y(-2) = 0$.



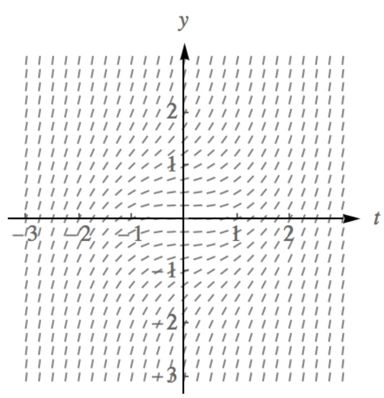
Example 3 (§7.9 Ex 45). *Match equations a-d with the direction fields A-D.*

a. $y'(t) = t/2$

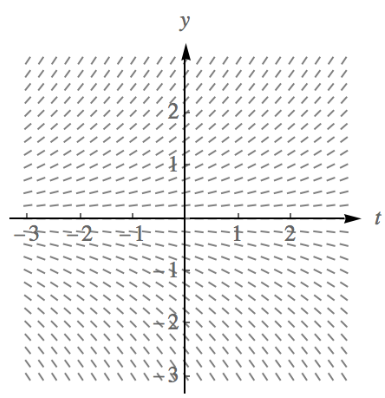
b. $y'(t) = y/2$

c. $y'(t) = (t^2 + y^2)/2$

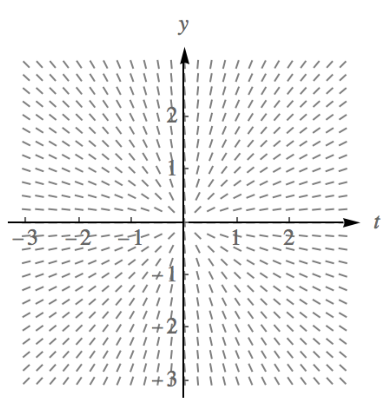
d. $y'(t) = y/t$



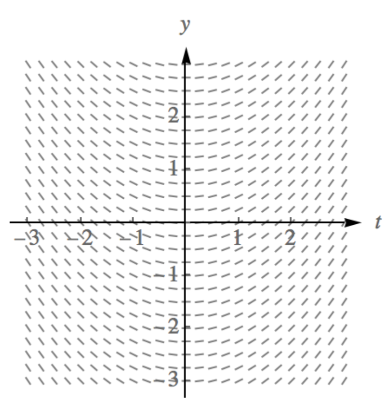
(A)



(B)



(C)



(D)