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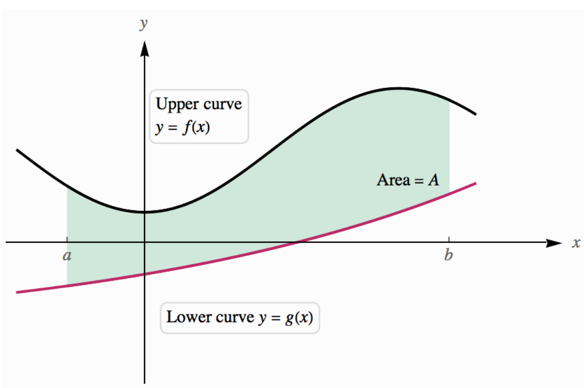
What is on today

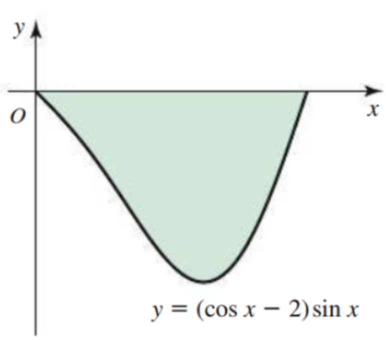
1 Regions between curves

Briggs-Cochran-Gillett-Schulz §6.2 pp. 412 - 420

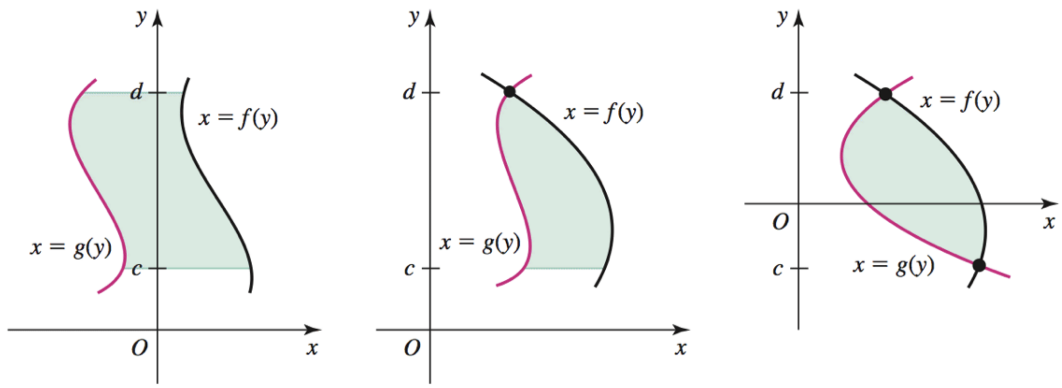
Definition 1. *Suppose that f and g are continuous functions with $f(x) \geq g(x)$ on the interval $[a, b]$. The area of the region bounded by the graphs of f and g on $[a, b]$ is $A = \int_a^b (f(x) - g(x))dx$.*



Example 2 (§6.2 Ex. 14)**.** *Determine the area of the shaded region:*



There are occasions when it is convenient to reverse the roles of x and y. Consider the regions shown below that are bounded by the graphs of $x = f(y)$ and $x = g(y)$, where $f(y) \geq g(y)$, for $c \leq y \leq d$ (which implies that the graph of f lies to the right of the graph of g). The lower and upper boundaries of the regions are $y = c$ and $y = d$, respectively.

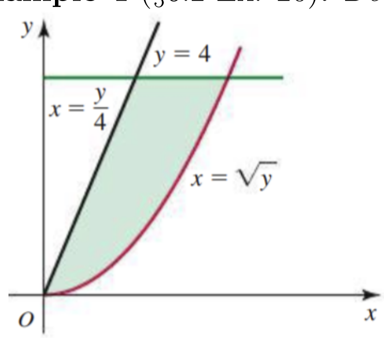


In cases like these, we treat y as the independent variable and divide the interval $[c, d]$ into n subintervals of width $\Delta y = (d - c)/n$. We compute the Riemann sum and take the limit to obtain the following:

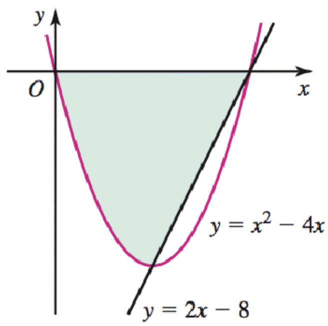
Definition 3. *Suppose that f and g are continuous functions with $f(y) \geq g(y)$ on the interval $[c, d]$. The area of the region bounded by $x = f(y)$ and $x = g(y)$ on $[c, d]$ is*

$$A = \int_c^d (f(y) - g(y))dy.$$

Example 4 (§6.2 Ex. 20)**.** *Determine the area of the following shaded region:*



Example 5 (§6.2 Ex. 32). *Express the area of the shaded region in terms of (a) one or more integrals with respect to x and (b) and one or more integrals with respect to y.*

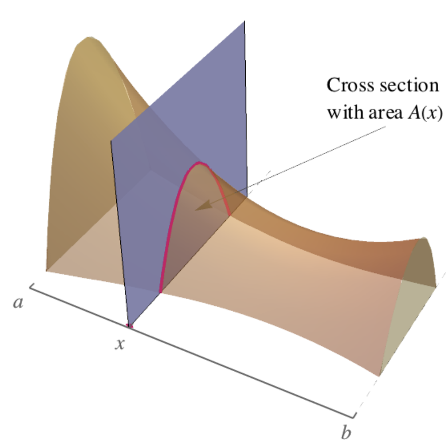


Example 6 (§6.2 Ex. 42). *Find the area of the region bounded by $y = 24\sqrt{x}$ and $y = 3x^2$*

2 Volume by slicing

Briggs-Cochran-Gillett-Schulz §6.3 pp. 425 - 429

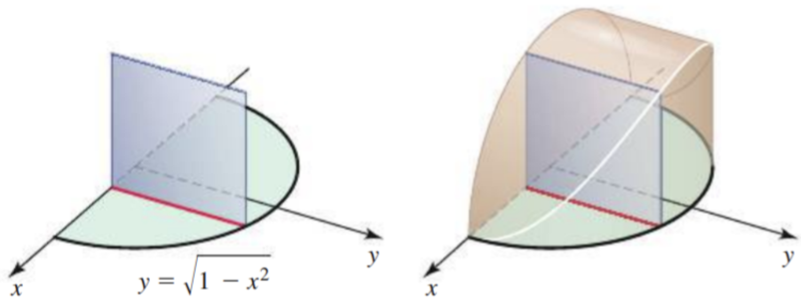
Consider a solid object that extends in the x-direction from $x = a$ to $x = b$. Imagine cutting through the solid, perpendicular to the x-axis at a particular point x, and suppose the area of the cross section is given by a function A.



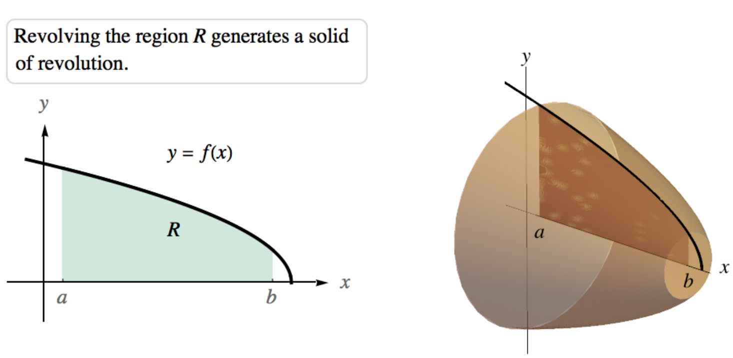
We give a formula to calculate the volume of this object, which is the basis of other volume formulas in this section:

> **General slicing method**
> Suppose a solid object extends from $x = a$ to $x = b$ and the cross section of the solid perpendicular to the x-axis has an area given by a function A that is integrable on $[a, b]$. The volume of the solid is
> $$V = \int_a^b A(x)dx.$$

Example 7 (§6.3 Ex. 11)**.** *Use the slicing method to calculate the volume of the solid whose base is the region bounded by the semicircle* $y = \sqrt{1 - x^2}$ *and the* x*-axis, and whose cross sections through the solid perpendicular to the* x*-axis are squares.*



We can also consider a specific type of solid known as a **solid of revolution**. Suppose f is a continuous function with $f(x) \geq 0$ on $[a, b]$. Let R be the region bounded by the graph of f, the x-axis, the the lines $x = a$ and $x = b$. Now rotate R around the x-axis. As R revolves once about the x-axis, it sweeps out a 3-dimensional solid of revolution.



We find the volume of this solid using the general slicing method. Here the cross-sectional area function has a special form since all cross-sections perpendicular to the x-axis are circular disks with radius $f(x)$. Thus the area of a cross-section is

$$A(x) = \pi(f(x))^2.$$

Here we summarize what just happened, which we call the disk method:

Disk method about the x-axis
Let f be continuous with $f(x) \geq 0$ on $[a, b]$. If the region R bounded by the graph of f, the x-axis, and the lines $x = a$ and $x = b$ is revolved about the x-axis, the volume of the resulting solid of revolution is

$$V = \int_a^b \pi(f(x))^2 dx.$$

Example 8 (§6.3 Ex. 18)**.** *Let R be the region bounded by the curves $y = 2-2x, y = 0, x = 0$. Use the disk method to find the volume of the solid generated when R is revolved about the x-axis. (What shape is this? Can you match this with a volume formula that you know?)*