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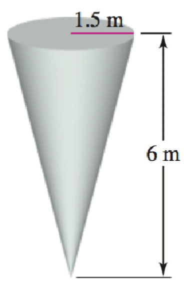
What is on today

1 Work wrap-up

Briggs-Cochran-Gillett-Schulz §6.7 pp. 465 - 471

Example 1 (§6.7 Ex. 39)**.** *A water tank is shaped like an inverted cone with height 6m and base radius 1.5 m (see figure).*



1. *If the tank is full, how much work is required to pump the water to the top of the tank and out of the tank?*
2. *Is it true that it takes half as much work to pump the water out of the tank when it is filled to half its depth as when it is full? Explain.*

Example 2 (§6.7 Ex. 31)**.** *A 30-m-long chain hangs vertically from a cylinder attached to a winch. Assume there is no friction in the system and the chain has a density of 5 kg/m.*

1. *How much work is required to wind the entire chain onto the cylinder using the winch?*

2. *How much work is required to wind the chain onto the cylinder if a 50-kg block is attached to the end of the chain?*

2 Exponential models

Briggs-Cochran-Gillett-Schulz §7.2 pp. 492 - 498

Exponential functions are used to model problems in a number of fields: finance, medicine, ecology, biology, physics, to name a few. In this section, we study exponential models.

Exponential growth functions have the form

$$y(t) = Ce^{kt},$$

where C is a constant and the rate constant k is positive. If we start with such a function and take its derivative, we find

$$y'(t) = kCe^{kt} = ky.$$

We see that the growth rate $y'(t)$ is proportional to the value of the function. Another interesting quantity to consider is the relative growth rate $y'(t)/y(t)$, which is constant for exponential functions. Note that the initial value $y(0) = C$ and the rate constant determine the exponential function completely.

The quantity described by the function $y(t) = y_0 e^{kt}$ for $k > 0$ has a constant doubling time of $T_2 = \frac{\ln 2}{k}$.

Exponential decay is described by functions of the form $y(t) = y_0 e^{-kt}$. The initial value of y is $y(0) = y_0$ and the rate constant $k > 0$ determines the rate of decay. Exponential decay is characterized by a constant relative decay rate. The constant half-life is $T_{1/2} = \frac{\ln 2}{k}$.

Example 3 (§7.2 Ex. 28)**.** *A drug is eliminated from the body at a rate of* 15% *per hour. After how many hours does the amount of drug reach 10% of the initial dose?*

Example 4 (§7.2 Ex. 35). *Uranium-238 (U-238) has a half-life of 4.5 billion years. Geologists find a rock containing a mixture of U-238 and lead, and determine that 85% of the original U-238 remains; the other 15% has decayed into lead. How old is the rock?*

3 Basic approaches to integration

Briggs-Cochran-Gillett-Schulz §8.1 pp. 520 - 523

In this section, we review some integration techniques. Here is a table of frequently used derivatives and antiderivatives:

$\frac{d}{du}F(u) = f(u)$	$\int f(u)\,du = F(u) + C$
$\frac{d}{du}u^{n+1} = (n+1)u^n$	$\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{du}\ln u = \frac{1}{u}$	$\int \frac{1}{u}\,du = \ln\lvert u\rvert + C$
$\frac{d}{du}\sin u = \cos u$	$\int \cos u\,du = \sin u + C$
$\frac{d}{du}\cos u = -\sin u$	$\int \sin u\,du = -\cos u + C$
$\frac{d}{du}\tan u = \sec^2 u$	$\int \sec^2 u\,du = \tan u + C$
$\frac{d}{du}\sec u = \sec u\ \tan u$	$\int \sec u\ \tan u\,du = \sec u + C$
$\frac{d}{du}e^u = e^u$	$\int e^u\,du = e^u + C$
	$\int \tan u\,du = \ln\lvert\sec u\rvert + C$
	$\int \sec u\,du = \ln\lvert\sec u + \tan u\rvert + C$
$\frac{d}{du}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}$	$\int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1}u + C$
$\frac{d}{du}\tan^{-1}u = \frac{1}{1+u^2}$	$\int \frac{1}{1+u^2}\,du = \tan^{-1}u + C$

Example 5 (§8.1 Ex. 27)**.** *Compute* $\int \frac{2-3x}{\sqrt{1-x^2}}dx$.

Example 6 (§8.1 Ex. 32)**.** *Evaluate* $\int_0^2 \frac{x}{x^2+4x+8}dx$.

Example 7 (§8.1 Ex. 34)**.** *Evaluate* $\int_2^4 \frac{x^2+2}{x-1}dx$.

Example 8 (§8.1 Ex. 75)**.** *Consider the region* R *bounded by the graph of* $f(x) = \sqrt{x^2+1}$ *and the* x*-axis on the interval* $[0, 2]$.

1. *Find the volume of the solid formed when* R *is revolved about the* x*-axis.*

2. *Find the volume of the solid formed when* R *is revolved about the* y*-axis.*