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What is on today

1 Basic approaches to integration, wrap-up

Briggs-Cochran-Gillett-Schulz §8.1 pp. 520 - 523

Here is another table of frequently used antiderivatives:

1. $\int k\,dx = kx + C$, k real

2. $\int x^p\,dx = \frac{x^{p+1}}{p+1} + C$, $p \neq -1$ real

3. $\int \cos ax\,dx = \frac{1}{a}\sin ax + C$

4. $\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$

5. $\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + C$

6. $\int \csc^2 ax\,dx = -\frac{1}{a}\cot ax + C$

7. $\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax + C$

8. $\int \csc ax \cot ax\,dx = -\frac{1}{a}\csc ax + C$

9. $\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$

10. $\int \frac{dx}{x} = \ln|x| + C$

11. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

12. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$, $a > 0$

13. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$, $a > 0$

14. $\int \tan ax\,dx = \frac{1}{a}\ln|\sec ax| + C$

15. $\int \cot ax\,dx = \frac{1}{a}\ln|\sin ax| + C$

16. $\int \sec ax\,dx = \frac{1}{a}\ln|\sec ax + \tan ax| + C$

17. $\int \csc ax\,dx = -\frac{1}{a}\ln|\csc ax + \cot ax| + C$

Example 1 (§8.1 Ex. 34)**.** *Evaluate* $\int_2^4 \frac{x^2+2}{x-1}dx$.

Example 2 (§8.1 Ex. 32)**.** *Evaluate* $\int_0^2 \frac{x}{x^2+4x+8} dx$.

Example 3. *Find the area of the surface generated when the curve* $y = \frac{1}{16}(e^{8x} + e^{-8x})$ *on the interval* $-2 \le x \le 2$ *is revolved about the* x*-axis.*

2 Integration by parts

Briggs-Cochran-Gillett-Schulz §8.2 pp. 525 - 529

The technique of integration by parts comes from reversing the product rule for derivatives:

Suppose that u, v are differentiable functions. Then

$$\int u dv = uv - \int v du$$

$$\int_a^b u(x)v'(x)dx = u(x)v(x)|_a^b - \int_a^b v(x)u'(x)dx.$$

The key is to figure out which function should be u and which one dv.

Example 4. *Evaluate* $\int \ln x \, dx$.

Example 5 (§8.2 Ex. 11). *Evaluate* $\int te^{6t}\,dt$.

Example 6 (§8.2 Ex. 18). *Evaluate* $\int x^9 \ln x\,dx$.

Example 7 (§8.2 Ex. 33). *Evaluate* $\int_0^{\pi} x \sin x\,dx$.

Example 8 (§8.2 Ex. 36). *Evaluate* $\int_0^{\ln 2} xe^x\,dx$.

3 Trigonometric integrals

Briggs-Cochran-Gillett-Schulz §8.3 pp. 532 - 536

Evaluating $\int \sin^m x \cos^n x \, dx$

- If m is odd and positive and n is real: split off $\sin x$, rewrite the resulting even power of $\sin x$ in terms of $\cos x$, and then use $u = \cos x$.
- If n is odd and positive and m is real: split off $\cos x$, rewrite the resulting even power of $\cos x$ in terms of $\sin x$ and then use $u = \sin x$.
- If m and n are both even, nonnegative integers: use half-angle formulas to transform the integrand into a polynomial in $\cos 2x$ and apply the preceding strategies once again to powers of $\cos 2x$ greater than 1.

Half-angle formulas

$$\sin^2 x = \frac{1 - \cos(2x)}{2} \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

Example 9 (§8.3 Ex 12). *Evaluate* $\int cos^4(2\theta)\, d\theta$.

Example 10 (§8.3 Ex 16). *Evaluate* $\int \sin^2\theta \cos^5\theta \, d\theta$.