Professor Jennifer Balakrishnan, *jbala@bu.edu*

What is on today

1 Trigonometric integrals 1

2 Trigonometric substitution 2

3 Partial fractions 3

1 Trigonometric integrals

Briggs-Cochran-Gillett-Schulz §8.3 pp. 532 - 536

Reduction formulas
Let n be a positive integer.

1. $\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
2. $\int \cos^n x\,dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
3. $\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx, n \neq 1$
4. $\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx, n \neq 1$

Integrals of $\tan x, \cot x, \sec x, \csc x$

$$\int \tan x\,dx = \ln|\sec x| + C \qquad \int \cot x\,dx = \ln|\sin x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C \qquad \int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

Evaluating $\int \tan^m x \sec^n x\,dx$

- If n is even: split off $\sec^2 x$, rewrite the remaining even power of $\sec x$ in terms of $\tan x$, and use $u = \tan x$.

- If m is odd: split off $\sec x \tan x$, rewrite the remaining even power of $\tan x$ in terms of $\sec x$, and use $u = \sec x$.
- If m is even and n is odd: rewrite the even power of $\tan x$ in terms of $\sec x$ to produce a polynomial in $\sec x$; apply reduction formula 4 to each term.

Example 1 (§8.3 Ex 38). *Evaluate* $\int \tan^5 \theta \sec^4 \theta \, d\theta$.

2 Trigonometric substitution

Briggs-Cochran-Gillett-Schulz §8.4 pp. 538 - 543

For integrals with an $a^2 - x^2$ term, we make the trigonometric substitution $x = a \sin \theta$; note that this gives $\theta = \sin^{-1}(x/a)$ for $-\pi/2 \leq \theta \leq \pi/2$.

Example 2 (§8.4 Ex. 10). *Evaluate* $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx$.

Example 3 (§8.4 Ex 13)**.** *Evaluate* $\int \frac{dx}{(16-x^2)^{1/2}}$.

3 Partial fractions

Briggs-Cochran-Gillett-Schulz §8.5 pp. 546 - 554

In this section, we use partial fraction decomposition to integrate certain rational functions. The idea behind it is that it is straightforward to integrate

$$\int \frac{1}{x-2} + \frac{2}{x+4}\,dx,$$

but it's not quite obvious how to integrate

$$\int \frac{3x}{x^2+2x-8}\,dx.$$

However, it turns out that

$$\frac{3x}{x^2+2x-8} = \frac{1}{x-2} + \frac{2}{x+4},$$

so if we are able to compute the decomposition on the RHS, we can integrate.

To get the decomposition, the first step is to factor the denominator on the LHS:

$$x^2+2x-8 = (x-2)(x+4).$$

So our goal is to write

$$\frac{3x}{x^2+2x-8} = \frac{A}{x-2} + \frac{B}{x+4},$$

and we need to solve for A and B.

Multiplying both sides of the above by $x^2 + 2x - 8$, we get

$$3x = A(x+4) + B(x-2),$$

and we compare the linear terms (terms with x) and constant terms (terms without x): we have

$$3x = Ax + Bx \Rightarrow 3 = A + B$$

and

$$0 = 4A - 2B.$$

Solving this system gives $A = 1$ and $B = 2$.

Example 4 (§8.5 Ex 24)**.** *Evaluate* $\int \frac{8}{(x-2)(x+6)}\,dx$.

Example 5 (§8.5 Ex 35)**.** *Evaluate* $\int \frac{x^2+12x-4}{x^3-4x}\,dx$.

Example 6 (§8.5 Ex 67)**.** *Find the area of the region bounded by the curve* $y = \frac{10}{x^2-2x-24}$*, the x-axis, and the lines* $x = -2$ *and* $x = 2$.