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What is on today

1 Separable differential equations

Briggs-Cochran-Gillett-Schulz §9.3 pp. 614 - 618

A **separable** differential equation is one that can be written as

$$g(y)y'(t) = h(t),$$

where the terms that involve y appear on one side of the equation separated from the terms that involve x. In general, we solve the separable equation $g(y)y'(t) = h(t)$ by integrating both sides of the equation with respect to t:

$$\int g(y)y'(t)dt = \int h(t)dt$$
$$\int g(y)dy = \int h(t)dt.$$

Thus by changing variables on the left side of the equation, the solution relies on evaluating two integrals.

Example 1 (§9.3 Ex. 19, 23, 26, 18, 30)**.** *Determine whether the following equations are separable. If so, solve the given initial value problem.*

1. $\frac{dy}{dt} = ty + 2, y(1) = 2$

2. $\frac{dy}{dx} = e^{x-y}, y(0) = \ln 3$

3. $ty'(t) = y(y+1), y(3) = 1$

4. $y'(t) = e^{ty}, y(0) = 1.$

5. $y'(t) = y^3 \sin t, y(0) = 1.$

A widely used model for population growth is the *logistic equation*

$$P'(t) = rP\left(1 - \frac{P}{K}\right),$$

where $P(t)$ is the population for $t \geq 0$ and $r > 0$ and $K > 0$ are given constants.

We can compute that the general solution of the equation is

$$P(t) = \frac{K}{1 + Ce^{-rt}},$$

where C is a constant. Computing the limit

$$\lim_{t \to \infty} P(t) = K,$$

we say K is the **equilibrium**, or **steady-state** solution. It is also called the **carrying capacity**. All curves in the general solution approach the carrying capacity.

Example 2 (§9.3 Ex. 40)**.** *When an infected person is introduced into a closed and otherwise healthy community, the number of people who contract the disease (in the absence of any intervention) may be modeled by the logistic equation*

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{A}\right), P(0) = P_0,$$

where k is a positive infection rate, A is the number of people in the community, and P_0 is the number of infected people at $t = 0$. (The model also assumes no recovery.)

1. *Find the solution of the initial value problem, for $t \geq 0$, in terms of k, A, and P_0.*

2. *For a fixed value of k and A, describe the long-term behavior of the solutions, for any P_0 with $0 < P_0 < A$.*