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What is on today

1 Properties of power series

Briggs-Cochran-Gillett-Schulz §11.2 pp. 722 - 726

Theorem 1 (Differentiating and integrating power series)**.** *Suppose the power series* $\sum c_k(x-a)^k$ *converges for* $|x-a|<R$ *and defines a function* f *on that interval.*

1. *Then* f *is differentiable (which implies continuous) for* $|x-a|<R$*, and* f' *is found by differentiating the power series for* f *term by term; that is,* $f'(x)=\sum kc_k(x-a)^{k-1}$*, for* $|x-a|<R$*.*

2. *The indefinite integral of* f *is found by integrating the power series for* f *term by term; that is,* $\int f(x)dx=\sum c_k\frac{(x-a)^{k+1}}{k+1}+C$*, for* $|x-a|<R$*, where* C *is an arbitrary constant.*

Example 2 (§11.2 Ex. 52, 54, 56)**.** *Find the power series representation for* g *centered at 0 by differentiating or integrating the power series for* f *(perhaps more than once). Give the interval of convergence for the resulting series.*

1. $g(x)=\frac{1}{(1-x)^3}$ *using* $f(x)=\frac{1}{1-x}$

2. $g(x) = \frac{x}{(1+x^2)^2}$ *using* $f(x) = \frac{1}{1+x^2}$

3. $g(x) = \ln(1 + x^2)$ *using* $f(x) = \frac{x}{1+x^2}$

Example 3 (§11.2 Ex, 58, 60)**.** *Find power series representations centered at 0 for the following functions using known power series. Give the interval of convergence for the resulting series.*

1. $f(x) = \frac{1}{1-x^4}$

2. $f(x) = \ln\sqrt{1-x^2}$

Example 4 (§11.2 Ex. 68)**.** *Find the function represented by the series* $\sum_{k=1}^{\infty} \frac{x^{2k}}{4^k}$ *and find the interval of convergence of the series.*

2 Taylor series

Briggs-Cochran-Gillett-Schulz §11.3 pp. 731 - 740

Suppose a function f has derivatives $f^{(k)}(a)$ of all orders at the point a. If we write the nth order Taylor polynomial for f centered at a and allow n to increase indefinitely, we get a power series – the Taylor series for f centered at a:

$$c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots + c_n(x-a)^n + \cdots = \sum_{k=0}^{\infty} c_k(x-a)^k,$$

where the coefficients are given by

$$c_k = \frac{f^{(k)}(a)}{k!}, \quad k = 0, 1, 2, \ldots.$$

The special case of a Taylor series centered at 0 is called a Maclaurin series.

For the Taylor series to be useful, we need to know two things: the values of x for which the Taylor series converges, and the values of x for which the Taylor series for f *equals* f. We will study the first issue now in a few examples. We will look at the second issue during the next class.

Example 5 (§11.3 Ex. 11, 12)**.** *For each of the following functions,*

(a) Find the first four nonzero terms of the Maclaurin series.

(b) Write the power series using summation notation.

(c) Determine the interval of convergence.

1. $f(x) = e^{-x}$

2. $f(x) = \cos(2x)$