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What is on today

1 Working with Taylor series

Briggs-Cochran-Gillett §11.4 pp. 742 - 747

We wrap up our study of Taylor series today. We now know the Taylor series for many familiar functions, and we have a number of new tools for working with power series. Here we wrap up some additional techniques that make use of what we've studied thus far.

Example 1 (§11.4 Ex. 16, 22)**.** *Evaluate the following limits using Taylor series.*

1. $\lim_{x\to 4} \frac{x^2-16}{\ln(x-3)}$

2. $\lim_{x\to\infty} x(e^{1/x}-1)$

Example 2 (§11.4 Ex. 31, 32)**.** *For each of the following functions,*

(a) Differentiate the Taylor series about 0 for the following functions.

(b) Identify the function represented by the differentiated series.

(c) Give the interval of convergence of the power series for the derivative.

1. $f(x) = \tan^{-1} x$

2. $f(x) = -\ln(1 - x)$

Example 3 (§11.4 Ex. 41)**.** *Use a Taylor series to approximate the definite integral*

$$\int_0^{.35} \tan^{-1} x \, dx.$$

Use as many terms as needed to ensure the error is less than 10^{-4}.

Example 4 (§11.4 Ex. 55, 62)**.** *Identify the functions represented by the following power series.*

1. $\sum_{k=0}^{\infty} \frac{x^k}{2^k}$

2. $\sum_{k=1}^{\infty} \frac{x^{2k}}{k}$

2 Review exercises: Taylor series

Briggs-Cochran-Gillett §11.R pp. 750 - 752

Example 5 (§11.R Ex. 3)**.** *Find the 2nd order Taylor polynomial for* $f(x) = \cos^3 x$ *centered at* $a = 0$*.*

Example 6 (§11.R Ex. 14)**.** *Find the remainder term* R_3 *for the Taylor series centered at 0 for the function* $f(x) = e^x$*. Find an upper bound for the magnitude of this remainder on the interval* $|x| < 1$*.*

Example 7 (§11.R Ex. 18)**.** *Determine the radius and interval of convergence of the power series* $\sum_{k=1}^{\infty} \frac{x^{4k}}{k^2}$.

Example 8 (§11.R Ex. 59)**.** *Use an appropriate Taylor series to find the first four nonzero terms of an infinite series that is equal to* $\sqrt{119}$.