Midterm 1 will cover the following sections in the textbook:
$\S\S 1.1-1.5, 1.7-1.10, 2.1-2.3, 2.5, 3.1-3.3$.

Here are some topics we have covered:

**Chapter 1. Linear Equations**

- Solving systems of linear equations.
- Elementary row operations and (Reduced) Row Echelon Form.
- Pivot positions, pivot columns.
- Rewriting a linear system as a matrix equation $A\mathbf{x} = \mathbf{b}$.
- Solutions of homogeneous equations $A\mathbf{x} = \mathbf{0}$.
- Solutions of the nonhomogeneous equation $A\mathbf{x} = \mathbf{b}$ are obtained by taking a particular solution $\mathbf{x}_0$ and adding all solutions of the homogeneous equation.
- Applications in business and science (§1.10).
- Linear independence of vectors.
- Linear transformations and their associated matrices, including geometric interpretations.
- Linear transformations being one-to-one and onto; some properties are below.

Let $A$ be an $m \times n$ matrix and $T : \mathbb{R}^n \to \mathbb{R}^m$ the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$.

| $T$ is one-to-one | $T$ is onto |
|---|---|
| $T(\mathbf{x}) = \mathbf{b}$ has *at most one solution* for every $\mathbf{b}$. | $T(\mathbf{x}) = \mathbf{b}$ has *at least one solution* for every $\mathbf{b}$. |
| The columns of $A$ are linearly independent. | The columns of $A$ span $\mathbb{R}^m$. |
| $A$ has a pivot in every column. | $A$ has a pivot in every row. |

**Chapter 2. Matrix Algebra**

- Addition and multiplication of matrices.
- The inverse of a square matrix.
- $A\mathbf{x} = \mathbf{b}$ has a unique solution if $A$ is invertible.
- $A \in M_{n\times n}$ is invertible iff its RREF is $I_n$. Know the algorithm for computing the inverse of a square matrix.
- The invertible matrix theorem: a square $n \times n$ matrix $A$ is invertible iff $A$ is one-to-one iff $A$ is onto. Be sure you know that $A$ is one-to-one iff the homogenous equation has only the trivial solution iff the columns of $A$ are linearly independent. Also, $A$ is onto iff $Ax = b$ has a solution for all $b$ iff $\mathrm{Col}(A)$ is all of $\mathbb{R}^n$.

- $(AB)^{-1} = B^{-1}A^{-1}$ if $A$ and $B$ are invertible. $(AB)^T = B^T A^T$.
- LU factorization.

**Chapter 3. Determinants**

- Computing the determinant of an $n \times n$ matrix using cofactors and using elementary row operations.
- $A$ is invertible iff $\det(A) \neq 0$.
- $(\det(A))(\det(B)) = \det(AB)$.
- Cramer's rule
- Inverse formula for a matrix involving determinant and adjugate
- For linear transformations $T : \mathbb{R}^2 \to \mathbb{R}^2$, we have area of $T(S)$ equals $|\det(A)|$ times the area of $S$ for reasonable sets $S$; there is a similar result for volumes for transformations on $\mathbb{R}^3$.